Petri nets — Exercise Sheet 6

Exercise 6.1

(a) Prove: If \((N, M_0)\) is a live S-system and \(M'_0 \geq M_0\), then \((N, M'_0)\) is also live.

(b) Prove: If \((N, M_0)\) is a live T-system and \(M'_0 \geq M_0\), then \((N, M'_0)\) is also live.

(c) Give an S-system \((N', M_0)\) that is 1-bounded and such that \(|M_0| > 1\).

(d) Give a strongly connected T-system \((N', M_0)\) which is not live and such that \(M_0 \neq 0\).

(e) Let \((N, M_0)\) be a T-system. Show that if \((N, M_0)\) is strongly connected and live, then it is bounded.

(f) Reprove (e), but this time without assuming that \((N, M_0)\) is live.

Exercise 6.2

(a) Show that the problem of determining whether a T-system is not live belongs to NP.

(b) Give a polynomial time algorithm for deciding liveness of T-systems.

(c) Test whether the following T-system is live by using your previous algorithm:

\[ \begin{array}{c}
\text{p}_1 \\
\text{p}_2 \\
\text{p}_3 \\
\text{p}_4 \\
\text{p}_5 \\
\text{p}_6 \\
\text{p}_7 \\
\text{p}_8 \\
\end{array} \]

Exercise 6.3

For each \(n \in \mathbb{N}\), give a 1-bounded T-system \((N, M_0)\) with \(n\) transitions and a reachable marking \(M\) such that the minimal occurrence sequence \(\sigma\) with \(M_0 \xrightarrow{\sigma} M\) has a length of \(\frac{n(n-1)}{2}\).

*Hint:* First try find a Petri net and a marking for \(n = 3\), where the minimal sequence has length 3. For this a net with 4 places suffices. Then try to generalize your solution.
Exercise 6.4
Consider the following free-choice system $(\mathcal{N}, M_0)$:

(a) Give all minimal proper siphons of $(\mathcal{N}, M_0)$.

(b) Use (a) to say whether $(\mathcal{N}, M_0)$ is live or not.