Technische Universität München **Summer Term 2019**
I7
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Petri nets — Exercise Sheet 6

Exercise 6.1

(a) Prove: If (N, M_0) is a live S-system and $M_0' \geq M_0$, then (N, M_0') is also live.

(b) Prove: If (N, M_0) is a live T-system and $M_0' \geq M_0$, then (N, M_0') is also live.

(c) Give an S-system $(\mathcal{N}, M_0)$ that is 1-bounded and such that $|M_0| > 1$.

(d) Give a strongly connected T-system $(\mathcal{N}, M_0)$ which is not live and such that $M_0 \neq \mathbf{0}$.

(e) Let $(\mathcal{N}, M_0)$ be a T-system. Show that if $(\mathcal{N}, M_0)$ is strongly connected and live, then it is bounded.

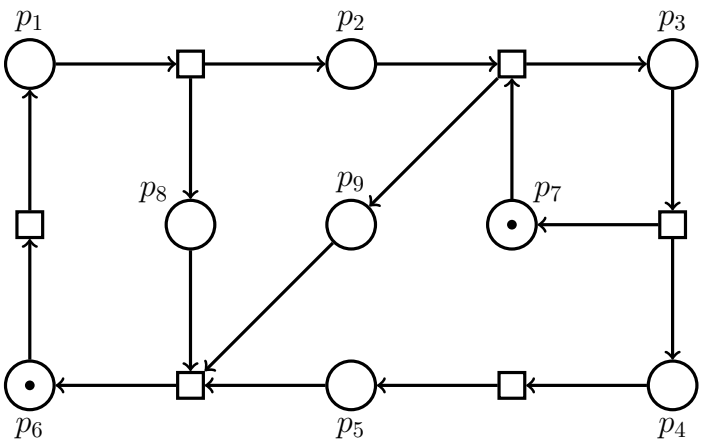
(f) Reprove (e), but this time without assuming that $(\mathcal{N}, M_0)$ is live.

Exercise 6.2

(a) Show that the problem of determining whether a T-system is *not live* belongs to NP.

(b) Give a polynomial time algorithm for deciding liveness of T-systems.

(c) Test whether the following T-system is live by using your previous algorithm:

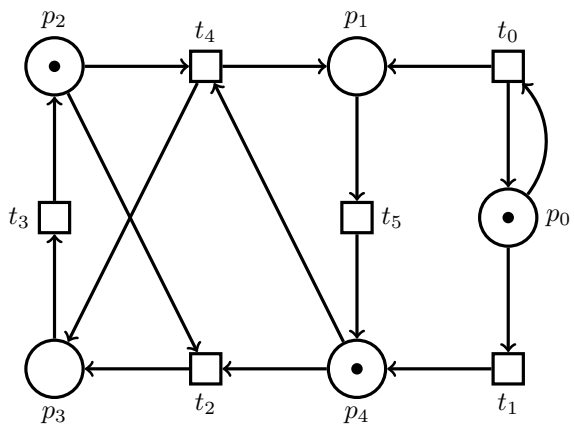


Exercise 6.3

For each $n \in \mathbb{N}$, give a 1-bounded T-system (N, M_0) with n transitions and a reachable marking M such that the minimal occurrence sequence σ with $M_0 \xrightarrow{\sigma} M$ has a length of $\frac{n(n-1)}{2}$.

Hint: First try find a Petri net and a marking for $n = 3$, where the minimal sequence has length 3. For this a net with 4 places suffices. Then try to generalize your solution.

Exercise 6.4
Consider the following free-choice system $(\mathcal{N}, M_0)$:



(a) Give all minimal proper siphons of $(\mathcal{N}, M_0)$.

(b) Use (a) to say whether $(\mathcal{N}, M_0)$ is live or not.