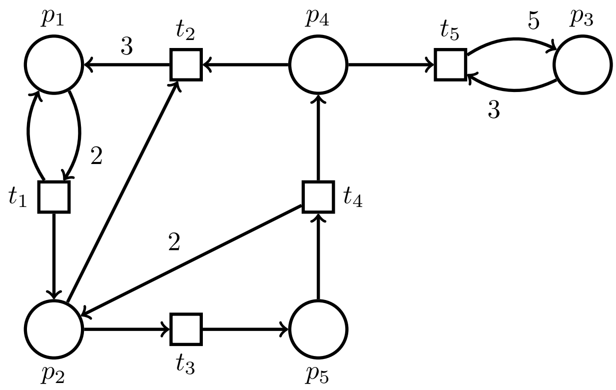
Petri nets — Exercise Sheet 5

Exercise 5.1
Consider the following Petri net (with weights) $\mathcal{N}$:



(a) Build the incidence matrix of $\mathcal{N}$.

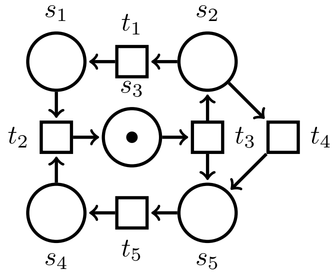
(b) Let $M_0 = \{p_1, p_1\}$. Try to determine whether

$$M_0 \xrightarrow{*} \{p_1, p_1, p_1, p_4\},$$
$$M_0 \xrightarrow{*} \{p_1, p_1, p_1, p_1, p_2\},$$
$$M_0 \xrightarrow{*} \{p_1, p_2, p_5\},$$

by solving the marking equation.

Exercise 5.2
For the following invariants, check if the net below has such an invariant. If yes, give one such invariant.

1. a semi-positive T-invariant
2. a positive T-invariant



Exercise 5.3
Exhibit counterexamples that disprove the following conjectures:

1. For a Petri net $(\mathcal{N}, M_0)$, an S-invariant I of $\mathcal{N}$ and a marking M, if $I \cdot M_0 = I \cdot M$, then M is reachable from M_0.
2. For a Petri net $(\mathcal{N}, M_0)$ and a place s of $\mathcal{N}$, if s is bounded, then there is an S-invariant I of $\mathcal{N}$ with $I(s) > 0$.

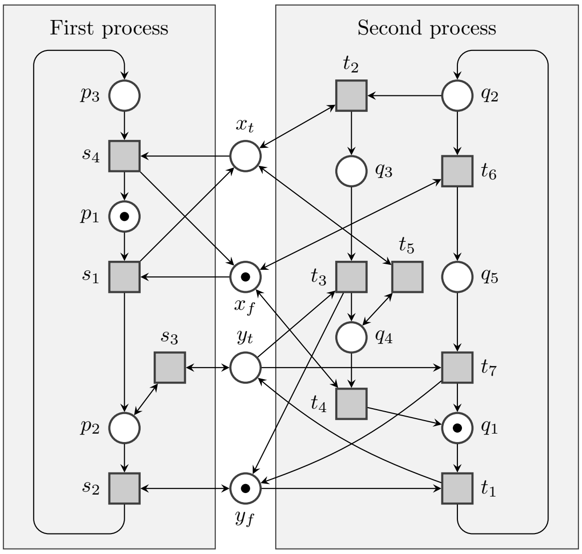
Exercise 5.4
On exercise sheet 1, we saw Lamport's algorithm for mutual exclusion. We used `LoLA` to show that it ensures mutual exclusion, which however needs state-space exploration. We now want to use invariants and traps to show that this holds. Below is a slightly smaller but equivalent model of Lamport's algorithm as a Petri net.

First process

```
     x = False
     while True:
p1:    x = True
p2:    while y: pass
p3:    # critical section
       x = False
```

Second process

```
     y = False
     while True:
q1:    y = True
q2:    if x:
q3:      y = False
q4:      while x: pass
         goto q1
q5:    # critical section
       y = False
```



The goal is to show that there is no reachable marking M such that $M(p_3) \geq 1$ and $M(q_5) \geq 1$.

(a) The net has the following S-invariants $I_1, \ldots, I_6$, which form a basis of the space of all S-invariants:

	p_1	p_2	p_3	x_t	x_f	q_1	q_2	q_3	q_4	q_5	y_t	y_f
I_1 =	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0
I_2 =	0	**1**	**1**	0	**1**	0	0	0	0	0	0	0
I_3 =	0	0	0	**1**	**1**	0	0	0	0	0	0	0
I_4 =	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	0	0
I_5 =	0	0	0	0	0	0	**1**	**1**	0	**1**	0	**1**
I_6 =	0	0	0	0	0	0	0	0	0	0	**1**	**1**

Use these invariants to show that there is a unique marking M where $M \sim M_0$, $M(p_3) \geq 1$ and $M(q_5) \geq 1$.

(b) Use traps to show that the marking M derived in (a) is not reachable from M_0. For this, find the largest unmarked trap at M using the algorithm for the largest siphon, adapted to traps.

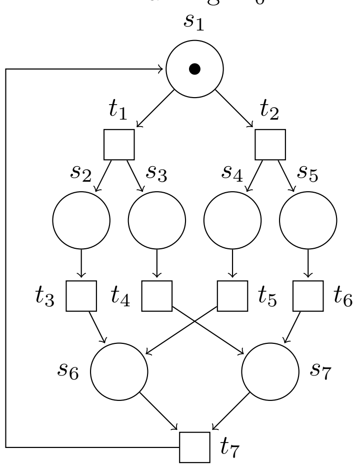
Exercise 5.5

1. Give a procedure that, given a net $\mathcal{N}$, constructs a boolean formula φ satisfying the following properties:
   - The formula contains variables r_s for each place $s \in S$,

- if φ is satisfiable, then $\mathcal{N}$ has a trap,
- and if φ is not satisfiable, then $\mathcal{N}$ has no trap.
- Additionally, if A is a model of φ, then the set given by $R = \{s \mid A(r_s)\}$ is a trap of $\mathcal{N}$.

2. Apply your procedure to the Petri net on the left below and give the resulting constraints.
3. Adapt your procedure such that, given two marking M_0 and M, it adds additional constraints to ensure that any trap R obtained as a solution by the constraints is marked at M_0 and unmarked at M. The constraints should be satisfiable iff a trap marked at M_0 and unmarked at M exists.
4. Construct the constraints for the Petri net below with the markings M_0 and M.
5. Use your constraints and the trap property to show that M is not reachable from M_0 in the net below.

Marking M_0



Marking M