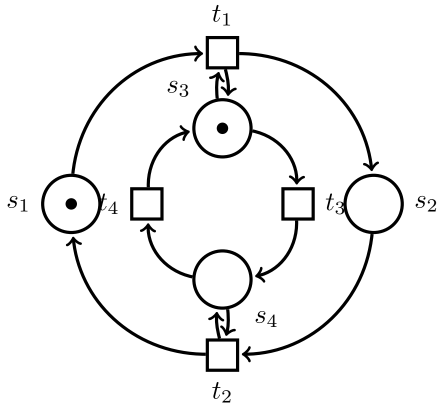
Petri nets — Exercise sheet 2

Exercise 2.1

(a) Give a net $\mathcal{N}$ and two markings M and M' such that $M \leq M'$, $(\mathcal{N}, M)$ is bounded, and $(\mathcal{N}, M')$ is *not* bounded.

(b) Give a net $\mathcal{N}$ and two markings M and M' such that $M \leq M'$, $(\mathcal{N}, M)$ is deadlock-free, and $(\mathcal{N}, M')$ is *not* deadlock-free.

(c) Give a net $\mathcal{N}$ and two markings M and M' such that $M \leq M'$, $(\mathcal{N}, M)$ is bounded *and* live, and $(\mathcal{N}, M')$ is unbounded. *Hint:* Add a place and arcs to the following net to obtain a solution:



Exercise 2.2

Let $\mathcal{N} = (P, T, W)$ be a net with weighted arcs. Let $M, M' \in \mathbb{N}^P$, $\sigma, \sigma' \in T^*$ and $t \in T$ be such that $M \xrightarrow{\sigma t \sigma'} M'$. Prove or disprove the following statements:

(a) if t does not consume any token, i.e $W(p,t) = 0$ for every $p \in P$, then $M \xrightarrow{t\sigma\sigma'} M'$.

(b) if t consumes no more tokens than it produces, i.e $W(p,t) \leq W(t,p)$ for every $p \in P$, then $M \xrightarrow{t\sigma\sigma'} M'$.

(c) if t produces no more tokens than it consumes, i.e. $W(t,p) \leq W(p,t)$ for every $p \in P$, then $M \xrightarrow{\sigma\sigma' t} M'$.

Exercise 2.3

Show that nets with place capacities and nets with weighted arcs are equivalent to standard nets. More precisely, sketch two algorithms solving the two following problems. The worst-case running time of your algorithms should be exponential.

(a)

INPUT: a net with place capacities $\mathcal{N} = (S, T, F, K)$, and two markings M and M'.

OUTPUT: a net $\mathcal{N}' = (S', T', F')$, and two markings L and L', such that $M \xrightarrow{*} M'$ in $\mathcal{N}$ if and only if $L \xrightarrow{*} L'$ in $\mathcal{N}'$.

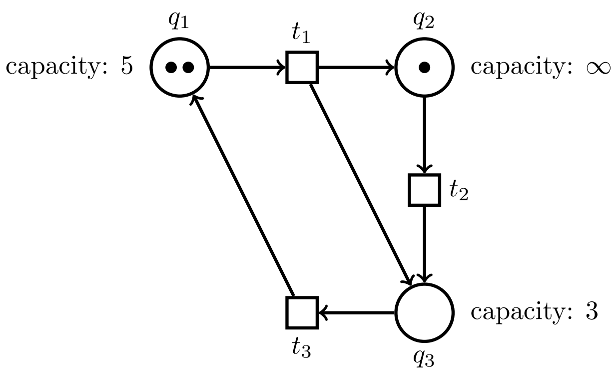
Apply your algorithm on the net below to the left with $M = \{2 \cdot q_1, q_2\}$ and $M' = \{2 \cdot q_1, q_3\}$.

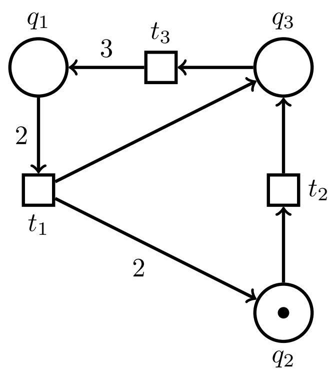
(b)

INPUT: a net with weighted arcs $\mathcal{N} = (S, T, W)$ and a markings M and M'.

OUTPUT: a net $\mathcal{N}' = (S', T', F')$, and two markings L and L', such that $M \xrightarrow{*} M'$ in $\mathcal{N}$ if and only if $L \xrightarrow{*} L'$ in $\mathcal{N}'$.

Apply your algorithm on the net below to the right with $M = \{q_2\}$ and $M' = \{q_1, 2 \cdot q_2, q_3\}$.





Exercise 2.4

We define *signed nets* as $\mathcal{N} = (S, T, F, \Sigma, s_\mathcal{N})$, where (S, T, F) is a classic net, Σ is a finite alphabet of *labels* and $s_\mathcal{N} : T \to \Sigma$ is called the *signature* of $\mathcal{N}$. Notice that the signature can assign the same label to two different transitions. The signature can be extended to a function $s_\mathcal{N}$ from T^* to Σ^*, using $s_\mathcal{N}(\sigma t) = s_\mathcal{N}(\sigma)s_\mathcal{N}(t)$ for $t \in T, \sigma \in T^*$. We call *free* a signed net such that $\Sigma = T$ and $s_\mathcal{N}$ is the identity.

For $M_0 \in \mathbb{N}^S$ a marking, we call $(\mathcal{N}, M_0)$ a *signed Petri net*. For $\mathcal{C}_t$ a set of markings, we call $(\mathcal{N}, M_0, \mathcal{C}_t)$ a *terminal signed Petri net*, and $\mathcal{C}_t$ is its set of *terminal markings*. We define

$$L_t(\mathcal{N}) = \{w \in \Sigma^* | \exists \sigma \in T^*, \exists C \in \mathcal{C}_t . M_0 \xrightarrow{\sigma} C \wedge s_\mathcal{N}(\sigma) = w\}$$

the *terminal language* for $(\mathcal{N}, M_0, \mathcal{C}_t)$.

(a) Show that regular languages (defined by automata) are included in terminal languages. *Hint:* Think of how to transform a DFA into a signed net.

(b) Terminal languages can express more than just regular languages. For instance, show that the non context-free language $\{a^n b^n c^n | n \in \mathbb{N}, n \geq 1\}$ is a terminal language for a certain terminal signed Petri net with a finite terminal set of markings.

(c) If we consider only *free* terminal signed Petri nets, that is terminal signed Petri nets without duplication of labels, then there are regular languages that are not terminal languages. There are even finite languages that are not terminal languages, for example $L = \{abc, ba\}$. Show that L is not a terminal language for any free terminal signed Petri net.

References

[1] Lutz Priese and Harro Wimmel. Petri-netze. ISBN 3-540-44289-8, 2003.