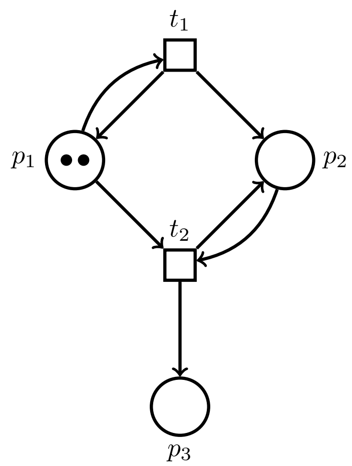
Petri nets — Endterm

- You have **75 minutes** to complete the exam.
- Answers must be written in a **separate booklet**. Do not answer on the exam.
- Please let us know if you need more paper.
- Write your name and Matrikelnummer on every sheet.
- Write with a non-erasable **pen**. Do not use red or green.
- You are not allowed to use auxiliary means other than pen and paper.
- You can obtain **40 points**. You need **17 points** to pass.
- Note that we sometimes represent a marking M by the tuple $(M(p_1), M(p_2), \ldots, M(p_n))$.

Question 1 (8 points)

Apply the backwards reachability algorithm to decide if the marking $M = (0, 0, 2)$ is coverable by the initial marking $M_0 = (2, 0, 0)$. Record all intermediate sets of markings with their finite representation of minimal elements.



Question 2 ($4 + 5 = 9$ points)

(a) Prove: Let (N, M_0) be a live T-system. For every marking M, if M is reachable from M_0, then M_0 is reachable from M.

(b) Consider a T-system (N, M_0). We say (N, M_0) is *floodable* if for all $k \in \mathbb{N}$, there exists a marking M reachable from M_0 with at least k tokens in each place, i.e. such that $M(s) \geq k$ for all $s \in S$.

Give an algorithm that runs in polynomial time and decides whether or not a given T-system (N, M_0) is floodable.

Question 3 (4 + 4 = 8 points)

Let (N, M_0) be a Petri net with $N = (S, T, F)$, and let M_t be a marking of N. We define

$$L(N, M_0, M_t) = \{w \in T^* | M_0 \xrightarrow{w} M_t\}$$

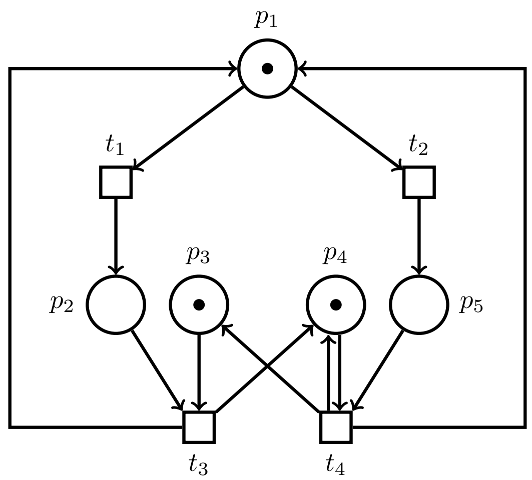
the *terminal language* for (N, M_0, M_t), where the transition set T is the alphabet of the language and the words are the transition sequences leading to the *terminal marking* M_t.

(a) Give a Petri net (N, M_0) and a terminal marking M_t such that the transition set of N is $T = \{a, b, c\}$ and the terminal language for (N, M_0, M_t) is $L = \{aba, aca\}$.

(b) Give a Petri net (N, M_0) such that the transition set of N is $T = \{a, b, c\}$ and the terminal language for (N, M_0, M_0) is $L = L((abcb)^*) = \{\epsilon, abcb, abcbabcb, \ldots\}$.

Question 4 (3 + 4 + 2 = 9 points)

Consider the following free-choice system:



(a) Give all minimal proper siphons of the net.
Hint: There are four minimal proper siphons.

(b) Which of the siphons of (a) contain a proper trap? Justify your answer by giving the traps if they contain one, or showing why no proper subset of the siphon is a trap.

(c) Use the results from (a) and (b) to decide if the system is live.

Question 5 (6 points)

Consider the class of Petri nets N where the following holds:

For all markings M, M' and vectors $X : T \to \mathbb{N}$, if $M' = M + \boldsymbol{N} \cdot X$ then there exists a sequence σ such that $\boldsymbol{\sigma} = X$ and $M \xrightarrow{\sigma} M'$

For this class of Petri nets, give an algorithm to decide the following problem by a reduction to the problem of deciding if a linear system of equations has an integer solution:

Given a system (N, M_0) and a transition t of N, is there an infinite run σ in (N, M_0) such that t occurs infinitely many times in σ?

The algorithm should construct a linear system of equations such that the system has an integer solution if and only if the answer to the problem is positive. Further, the reduction should run in polynomial time.