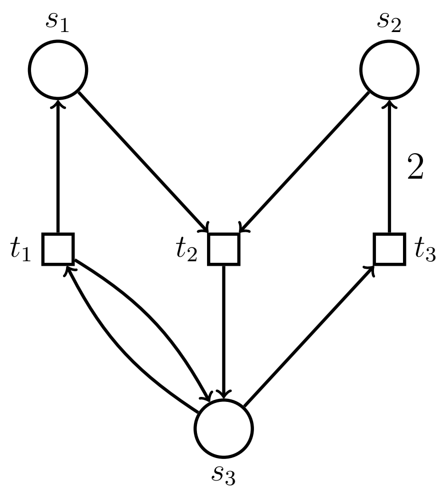
Petri nets — Endterm

- You have **90 minutes** to complete the exam.
- Answers must be written in a **separate booklet**. Do not answer on the exam.
- Please let us know if you need more paper.
- Write your name and Matrikelnummer on every sheet.
- Write with a non-erasable **pen**. Do not use red or green.
- You are not allowed to use auxiliary means other than pen and paper.
- You can obtain **40 points**. You need **17 points** to pass.
- Note that we sometimes represent a marking M by the tuple $(M(s_1), M(s_2), \ldots, M(s_n))$.

Question 1 (4 + 2 = 6 points)
Consider the following Petri net with weights $N = (S, T, W)$:



Let $M_0 = (0, 1, 0)$ and $M = (1, 0, 1)$. We wish to determine whether M is coverable from M_0. After one iteration of the backward reachability algorithm, we obtain the minimal basis $X = \{(2, 1, 0), (0, 0, 1)\}$.

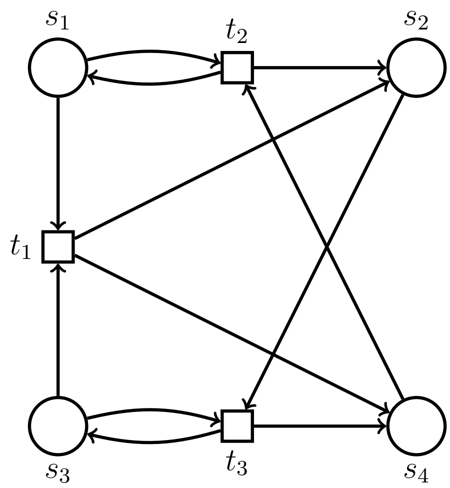
(a) Give the *minimal* basis Y obtained by executing the next iteration of the backward reachability algorithm from X.

(b) What can you conclude from Y obtained in (a)?

1. M is coverable from M_0.
2. M is not coverable from M_0.
3. None of the above, another iteration must be executed.

Justify your answer.

Question 2 (2 + 2 + 2 + 2 = 8 points)
Consider the following Petri net $N = (S, T, F)$:



(a) Give all of the minimal proper traps of N. Explain briefly why no other proper trap is minimal.

(b) Does N have a positive S-invariant? If so, exhibit one, if not, explain why.

(c) Prove that $M = (5, 13, 7, 15)$ is *not* reachable from $M_0 = (15, 3, 17, 4)$.

(d) Prove that $M = (0, 20, 0, 0)$ is *not* reachable from $M_0 = (10, 0, 10, 0)$.

Question 3 (5 points)
Recall that 3-SAT is the problem of determining the satisfiabillity of a Boolean formula in conjunctive normal form where clauses have at most three literals.

Give a polynomial time reduction from 3-SAT to the following reachability problem for 1-safe Petri nets:

Given:	1-safe Petri net (N, M_0) and a place s of N.
Determine:	does there exist a marking M such that $M_0 \xrightarrow{*} M$ and $M(s) = 1$?

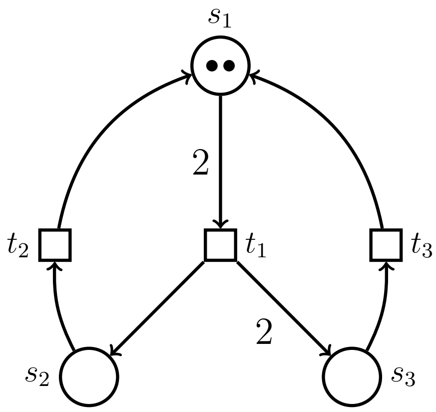
It suffices to explain your reduction informally and to illustrate it for the following formula:

$$\varphi(x_1, x_2, x_3, x_4) = (x_1 \vee \neg x_2 \vee x_4) \wedge (\neg x_1 \vee x_3 \vee x_4).$$

Question 4 (4 points)
The *projection* of a firing sequence $\sigma \in T^*$ onto $U \subseteq T$ is the sequence $h_U(\sigma)$ obtained by deleting all transitions of σ which do not belong to U. For example, $h_{\{u,v\}}(uvtvt) = uvv$, $h_{\{t\}}(uvtvt) = tt$ and $h_{\{u,v\}}(ttt) = \varepsilon$. The *$U$-traces* of a Petri net (N, M_0) is the set

$$L_U(N, M_0) = \{h_U(\sigma) : \sigma \text{ is a firing sequence enabled at } M_0\}.$$

Consider the following deadlock-free Petri net with weights (N, M_0):



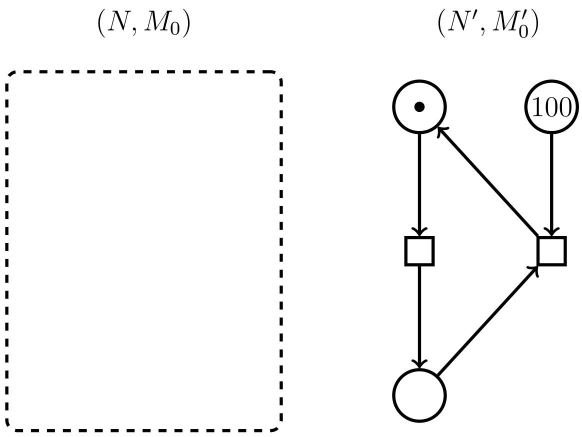
Give a new Petri net (N', M'_0) such that

- N' has *no weights*,
- (N', M'_0) is deadlock-free, and
- $L_{\{t_1,t_2,t_3\}}(N', M'_0) = L_{\{t_1,t_2,t_3\}}(N, M_0)$.

Question 5 (3 + 3 + 3= 9 points)

(a) Give a Petri net (N, M_0) and connect it with arcs to the Petri net (N', M'_0) shown below so that the resulting Petri net (N'', M''_0) satisfies the following properties:

- (N'', M''_0) is bounded, and
- (N'', M''_0) has a reachable marking with at least 2^{100} tokens.



(b) Exhibit a connected Petri net $N = (S, T, F)$ such that $I = (1, -1, 0)$ is an S-invariant and $J = (1, 0, 1)$ is a T-invariant of N. Justify your answer.

(c) Exhibit a deadlock-free Petri net (N, M_0) and a marking $M \geq M_0$ such that (N, M) is *not* deadlock-free.

Question 6 (4 + 4 = 8 points)

(a) Let (N, M_0) be a live T-system. Prove that (N, M_0) is cyclic.

(b) Let N be a T-net and let M_0, M be markings. Prove that if (N, M_0) is live and $2M_0 \xrightarrow{*} 2M$, then $M_0 \xrightarrow{*} M$.