Petri nets — Homework 6

Due 27.06.2018

Exercise 6.1

(a) Give an S-system \((\mathcal{N}, M_0)\) that is 1-bounded and such that \(|M_0| > 1\).

(b) Give a strongly connected T-system \((\mathcal{N}, M_0)\) which is not live and such that \(M_0 \neq 0\).

(c) Give a bounded T-system \((\mathcal{N}, M_0)\) which is not strongly connected and such that \(M_0 \neq 0\).

(d) Let \((\mathcal{N}, M_0)\) be a T-system. Show that if \((\mathcal{N}, M_0)\) is strongly connected and live, then it is bounded.

(e) ★ Reprove (d), but this time without assuming that \((\mathcal{N}, M_0)\) is live.

Exercise 6.2

(a) Let \(\mathcal{N} = (P, T, F)\) be a Petri net, and let \(s, t \in T\) be such that \(s \cap t^* = \emptyset\). Show that if \(M \xrightarrow{ts} M'\), then \(M \xrightarrow{st} M'\).

(b) Let \(\mathcal{N} = (P, T, F)\) be a Petri net which is not strongly connected. Show that \(P \cup T\) can be partitioned into two disjoint sets \(U, V \subseteq P \cup T\) such that \(F \cap (V \times U) = \emptyset\).

(c) Let \(U\) and \(V\) be a partition as in (b). Show that if \(M \xrightarrow{UV} M'\), then there exist \(\sigma_U \in (T \cap U)^*\) and \(\sigma_V \in (T \cap V)^*\) such that \(\sigma = \sigma_U \sigma_V\) and \(M \xrightarrow{\sigma_U \sigma_V} M'\).

(d) Let \((\mathcal{N}, M_0)\) be live and bounded. Use (a), (b) and (c) to show that \(\mathcal{N}\) is strongly connected.

Exercise 6.3

(a) Show that the problem of determining whether a T-system is not live belongs to NP.

(b) Give a polynomial time algorithm for deciding liveness of T-systems.

(c) Test whether the following T-system is live by using your previous algorithm:

\begin{center}
\begin{tikzpicture}[node distance=1.5cm, on grid, auto, scale=0.7, transform shape]
  \node[place] (p0) at (0,0) [color=black] {\(p_0\)}; 
  \node[place] (p1) at (2,0) [color=black] {\(p_1\)}; 
  \node[place] (p2) at (2,2) [color=black] {\(p_2\)}; 
  \node[place] (p3) at (4,0) [color=black] {\(p_3\)}; 
  \node[place] (p4) at (4,-2) [color=black] {\(p_4\)}; 
  \node[place] (p5) at (1,1) [color=black] {\(p_5\)}; 
  \node[place] (p6) at (3,1) [color=black] {\(p_6\)}; 
  \node[place] (p7) at (3,-1) [color=black] {\(p_7\)}; 
  \node[place] (p8) at (1,-1) [color=black] {\(p_8\)}; 
  \node[place] (p9) at (3,0) [color=black] {\(p_9\)}; 

  \draw (p0) edge [->] (p1); 
  \draw (p1) edge [->] (p2); 
  \draw (p2) edge [->] (p3); 
  \draw (p3) edge [->] (p4); 
  \draw (p4) edge [->] (p0); 
  \draw (p0) edge [loop above] (p0); 
  \draw (p1) edge [loop above] (p1); 
  \draw (p2) edge [loop above] (p2); 
  \draw (p3) edge [loop above] (p3); 
  \draw (p4) edge [loop above] (p4); 
  \draw (p5) edge [->] (p6); 
  \draw (p6) edge [->] (p7); 
  \draw (p7) edge [->] (p8); 
  \draw (p8) edge [->] (p9); 
  \draw (p9) edge [->] (p5); 
  \draw (p5) edge [loop below] (p5); 
  \draw (p6) edge [loop below] (p6); 
  \draw (p7) edge [loop below] (p7); 
  \draw (p8) edge [loop below] (p8); 
  \draw (p9) edge [loop below] (p9);
\end{tikzpicture}
\end{center}
Exercise 6.4
Consider the following free-choice system \((\mathcal{N}, M_0)\):

(a) Give all minimal proper siphons of \((\mathcal{N}, M_0)\).

(b) Use (a) to say whether \((\mathcal{N}, M_0)\) is live or not.