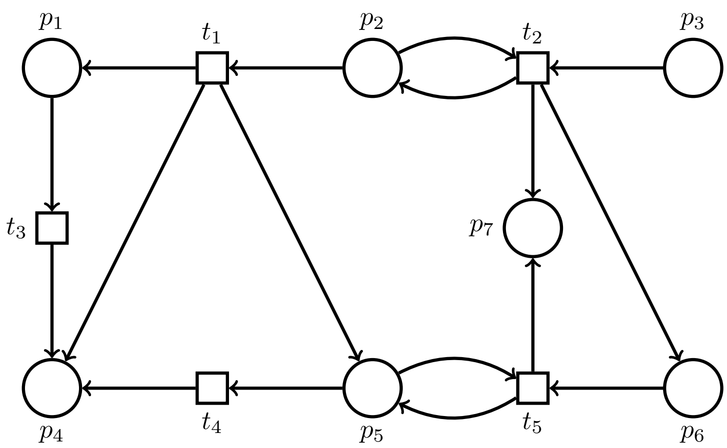
Technische Universität München **Summer term 2018**
I7
Prof. J. Esparza / Dr. M. Blondin **07.06.2018**

Petri nets — Homework 5

Due 13.06.2018

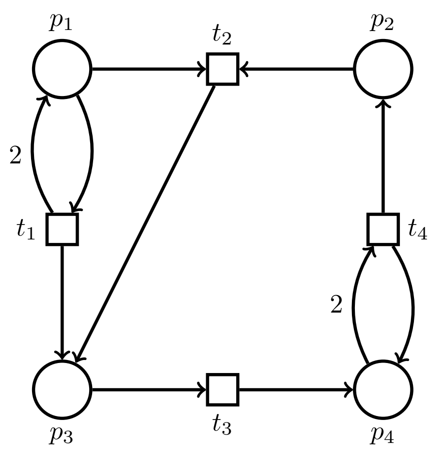
Exercise 5.1
Consider the following Petri net $\mathcal{N} = (P, T, F)$:



(a) Give a basis of the vector space of T-invariants of $\mathcal{N}$. [Hint:]

(b) Let $M = \{p_1, p_2, p_4, p_4\}$ and $M' = \{p_1, p_3, p_5\}$. We have shown that $\mathcal{N}$ is bounded from any initial marking in #4.3(b). Can you tell whether $(\mathcal{N}, M)$ and $(\mathcal{N}, M')$ are live?

Exercise 5.2
Consider the following Petri net (with weights) $\mathcal{N}$:



(a) Use siphons/traps to prove or disprove that $\mathcal{N}$ is live from $M_0 = \{p_2, 3 \cdot p_4\}$,

(b) Can the marking equation be used to prove or disprove that $\{p_2, p_4\} \xrightarrow{*} \{p_1, 3 \cdot p_2, p_3\}$? Is so, why? If not, can traps or siphons help?

Exercise 5.3

Let $\mathcal{N} = (P, T, W)$ be a net with weights and let M_0, M be markings. We say that a trap Q is *minimal* if it is non empty and every non empty $Q' \subset Q$ is not a trap.

(a) Disprove the following statement:

> There exists a trap Q of $\mathcal{N}$ such that $M_0(Q) > 0$ and $M(Q) = 0$ if and only if there exists a minimal trap R of $\mathcal{N}$ such that $M_0(R) > 0$ and $M(R) = 0$.

(b) Show that Petri nets can have exponentially many minimal traps. More formally, exhibit an infinite family of (distinct) nets $\mathcal{N}_1 = (P_1, T_1, F_1), \mathcal{N}_2 = (P_2, T_2, F_2), \ldots$, some $c > 1$, and a function $f \in \Omega(c^n)$ such that $\mathcal{N}_i$ has $f(|P_i|)$ minimal traps for every $i \geq 1$.

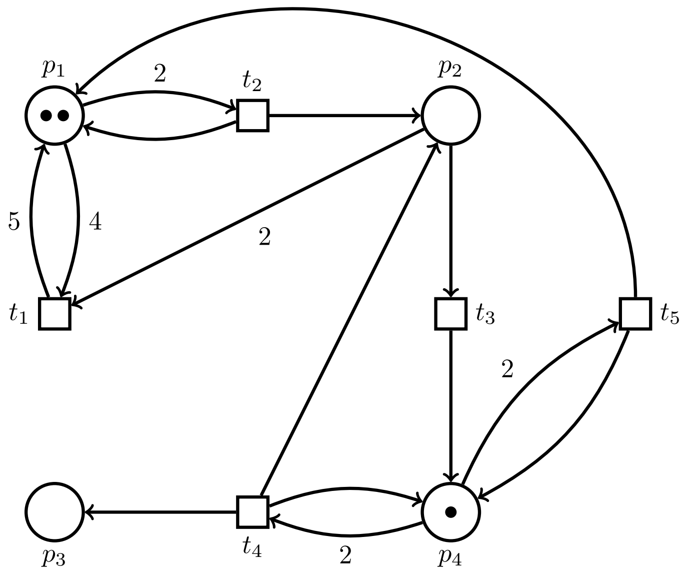
(c) For every $x \in \{\texttt{false}, \texttt{true}\}^P$, let $Q_x = \{p : p \in P, x_p = \texttt{true}\}$.

(i) Give a Boolean formula φ_{trap} over variables $\{x_p : p \in P\}$ such that $\varphi_{\text{trap}}(x)$ holds if and only if Q_x is a trap of $\mathcal{N}$.

(ii) Give a quantified Boolean formula φ_{mintrap} over variables $\{x_p : p \in P\}$ such that $\varphi_{\text{mintrap}}(x)$ holds if and only if Q_x is a minimal trap of $\mathcal{N}$.

(iii) Give a Boolean formula $\varphi_{\text{constraints}}$ over variables $\{x_p : p \in P\}$ such that $\varphi_{\text{constraints}}(x)$ holds if and only if $M_0(Q_x) > 0$ implies $M(Q_x) > 0$.

(d) Construct $\varphi_{\text{trap}}(x)$ for the following Petri net:



(e) ★ Use the SMT solver `Z3` to prove that the Petri net above cannot reach a marking M such that $M(p_2) = 2$ and $M(p_3) \geq 1$. See instructions on the webpage of the course. You may start from the given partial solution if you need help.