Exercise 4.1

(a) Show that
\[ X = \{ (x_1, x_2, x_3) \in \mathbb{N}^3 : (x_1 + 3 \leq x_2 \leq x_3 + 1) \lor (x_2 = 2x_1 + x_3 + 5) \} \]
is semilinear by giving its representation as a finite set of roots and periods.

(b) Give a Petri net whose reachability set equals \( X \) up to a projection. More precisely, give a Petri net (with weights) \( \mathcal{N} = (P, T, W) \) such that \( \{p_{\text{init}}, p_1, p_2, p_3\} \subseteq P \) and
\[ \{p_{\text{init}}\} \rightarrow M \text{ and } M(p_{\text{init}}) = 0 \iff (M(p_1), M(p_2), M(p_3)) \in X. \]

Exercise 4.2

Consider the following Petri net (with weights) \( \mathcal{N} \):

(a) Build the incidence matrix of \( \mathcal{N} \).

(b) Let \( M_0 = \{p_1, p_1\} \). Try to determine whether
\[ M_0 \rightarrow \{p_1, p_1, p_1, p_4\}, \]
\[ M_0 \rightarrow \{p_1, p_1, p_1, p_2\}, \]
\[ M_0 \rightarrow \{p_1, p_2, p_3\}, \]
by solving the marking equation.

(c) Does \( \{p_1, p_3\} \rightarrow \{p_2, p_2, p_2, p_4\} \)? Prove your answer.
Exercise 4.3
Consider the following Petri net $\mathcal{N} = (P, T, F)$:

(a) Give a basis of the vector space of $S$-invariants of $\mathcal{N}$. [Hint: ]

(b) Let $M = \{p_1, p_2, p_4, p_4\}$ and $M' = \{p_1, p_3, p_5\}$. Using (a), can you tell whether $(\mathcal{N}, M)$ and $(\mathcal{N}, M')$ are bounded? live?