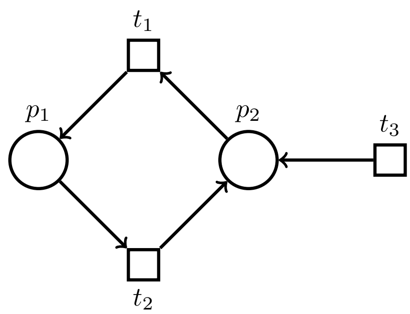
Petri nets — Homework 3

Due 23.05.2018

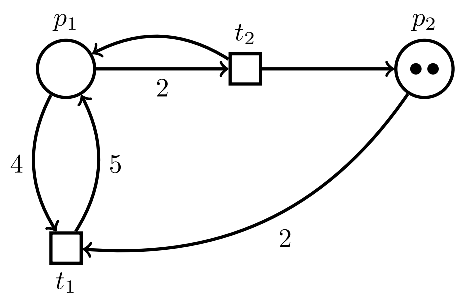
Exercise 3.1

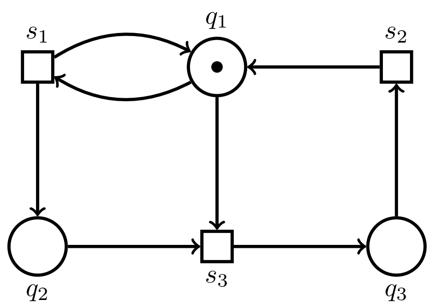
The algorithm COVERABILITY-GRAPH does not specify how the coverability graph should be traversed during its construction. Show that different traversal strategies can lead to different coverability graphs. More precisely, exhibit a marking M and two different coverability graphs for $(\mathcal{N}, M)$, where $\mathcal{N}$ is the following net:



Exercise 3.2

Let $\mathcal{N}$ and $\mathcal{N}'$ be respectively the left and right Petri nets below.





Use the backward algorithm to answer the following questions.

(a) Describe the set of initial markings from which $\{p_2, p_2\}$ is coverable in $\mathcal{N}$. Illustrate this set.

(b) Determine whether $\{q_1, q_3\}$ is coverable from $\{q_1\}$ in $\mathcal{N}'$.

(c) Determine whether $\{q_1, q_2\}$ is coverable from $\{q_1\}$ in $\mathcal{N}'$.

Exercise 3.3

A net with reset, doubling and transfer arcs is a tuple (P, T, F, R, D, Tr) where (P, T, F) is a net,

$$R \subseteq P \times T, \ D \subseteq T \times P, \ Tr \subseteq (P \times T) \cup (T \times P),$$

and F, R, D and Tr are pairwise disjoint. Let $M \in \mathbb{N}^P$ and $t \in T$. We say that t is enabled at M if $M(p) > 0$ for every $(p, t) \in F$. Firing t at M has the following effect:

- every arc $(p, t) \in F$ consumes a token from p;
- every arc $(t, p) \in F$ produces a token in p;
- every arc $(p, t) \in R$ empties p;
- every arc $(t, p) \in D$ doubles the amount of tokens in p;
- every arc $(p, t) \in Tr$ empties p;
- every arc $(t, p) \in Tr$ adds $\sum_{(q,t)\in Tr} M(q)$ tokens to p.

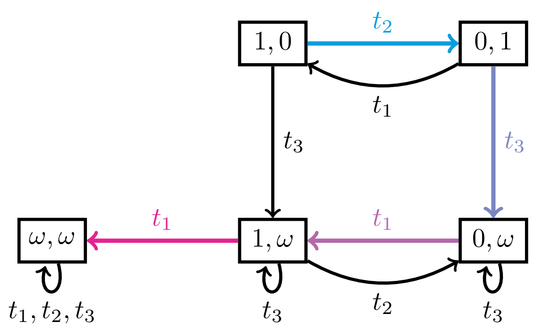
Show that the backward algorithm works for this extended class of nets by showing that it is monotonic, i.e. show that for every markings $X, X', Y \in \mathbb{N}^P$, if $X \to Y$ and $X' \geq X$, then $X' \to Y'$ for some $Y' \geq Y$.

Exercise 3.4

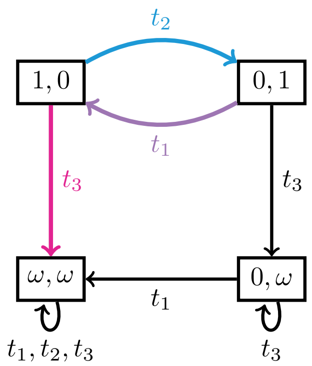
(a) Show that nets with inhibitor arcs are not monotonic.

(b) Give a net with reset arcs $\mathcal{N}$ and a marking M such that $(\mathcal{N}, M)$ is bounded, but such that there exists a sequence $M \xrightarrow{\sigma} M' \xrightarrow{\sigma'} M''$ with $M'' \geq M'$ and $M'' \neq M'$.

Solution 3.1
Let $M = \{p_1\}$. We exhibit two coverability graphs for $(\mathcal{N}, M)$, where nodes are labeled with respect to the total order $p_1 < p_2$. We construct the first coverability graph by first exploring the path $t_2 t_3 t_1 t_1$:



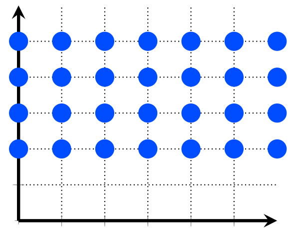
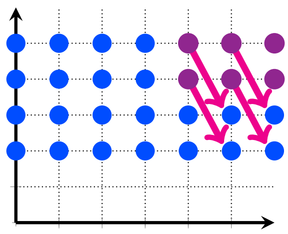
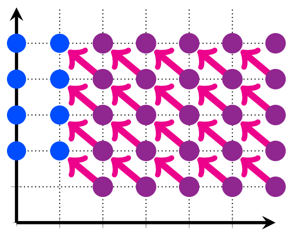
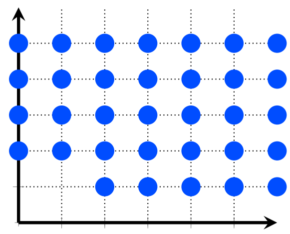
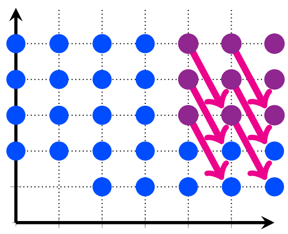
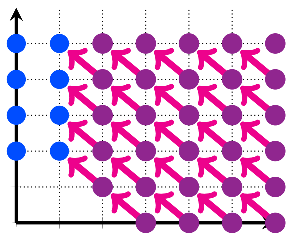
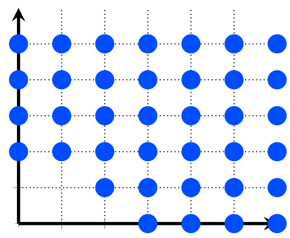
For the second coverability graph, we first explore the path $t_2 t_1 t_3$:



Note that the subprocedure AddOmegas generates (ω, ω) after exploring $t_2 t_1 t_3$ because, at this point, both $(1, 0)$ and $(0, 1)$ are "ancestors" of the current node labeled by $(1, 1)$.

Solution 3.2

(a) We execute the backward algorithm from $M = (0, 2)$. In order to build the whole set of initial markings, we ignore the stopping criterion based on M_0.

Iteration	$\text{pre}_{t_1}(m)$	$\text{pre}_{t_2}(m)$	m
0	—	—	
1			
2			
3	sets left unchanged		

The set of initial markings is $\{M \in \mathbb{N}^2 : M \geq (0, 2) \text{ or } M \geq (2, 1) \text{ or } M \geq (3, 0)\}$.

(b) We want to determine whether $M = (1,0,1)$ is coverable from $M_0 = (1,0,0)$. It is not the case, since executing the backward algorithm from M does not generate any marking less or equal to M_0:

Iteration	pre(m)	m
0	—	$\{(1,0,1)\}$
1	$\text{pre}_{s_1}(1,0,1) = (1,0,1)$ $\text{pre}_{s_2}(1,0,1) = (0,0,2)$ $\text{pre}_{s_3}(1,0,1) = (2,1,0)$	$\{(1,0,1),(0,0,2),(2,1,0)\}$
2	$\text{pre}_{s_1}(1,0,1) = (1,0,1)$ $\text{pre}_{s_2}(1,0,1) = (0,0,2)$ $\text{pre}_{s_3}(1,0,1) = (2,1,0)$ $\text{pre}_{s_1}(0,0,2) = (1,0,2)$ $\text{pre}_{s_2}(0,0,2) = (0,0,3)$ $\text{pre}_{s_3}(0,0,2) = (1,1,1)$ $\text{pre}_{s_1}(2,1,0) = (2,0,0)$ $\text{pre}_{s_2}(2,1,0) = (1,1,1)$ $\text{pre}_{s_3}(2,1,0) = (3,2,0)$	$\{(1,0,1),(0,0,2),(2,0,0)\}$
3	$\text{pre}_{s_1}(2,0,0) = (2,0,0)$ $\text{pre}_{s_2}(2,0,0) = (1,0,1)$ $\text{pre}_{s_3}(2,0,0) = (3,1,0)$	$\underbrace{\{(1,0,1),(0,0,2),(2,0,0)\}}_{\text{unchanged}}$

(c) We want to determine whether $M' = (1,1,0)$ is coverable from $M_0 = (1,0,0)$. It is the case, since executing the backward algorithm from M' yields M_0 after one iteration:

Iteration	pre(m)	m
0	—	$\{(1,1,0)\}$
1	$\text{pre}_{s_1}(1,1,0) = (1,0,0)$ $\text{pre}_{s_2}(1,1,0) = (0,1,1)$ $\text{pre}_{s_3}(1,1,0) = (2,2,0)$	$\{\underbrace{\mathbf{(1,0,0)}}_{\leq M_0},(0,1,1)\}$

Solution 3.3

Let $X, X', Y \in \mathbb{N}^P$ and $t \in T$ be such that $X \xrightarrow{t} Y$ and $X' \geq X$. Let us first argue that t is enabled at X':

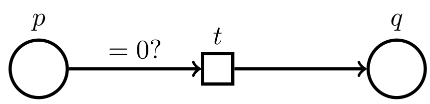
$$\begin{aligned} t \text{ is enabled at } X &\iff X(p) > 0 \text{ for every } (p,t) \in F \\ &\implies X'(p) > 0 \text{ for every } (p,t) \in F \qquad \text{(since } X' \geq X) \\ &\iff t \text{ is enabled at } X'. \end{aligned}$$

Let $Y' \in \mathbb{N}^P$ be the marking such that $X' \xrightarrow{t} Y'$. Let $p \in P$. It remains to show that $Y'(p) \geq Y(p)$. We only prove the case where p is not in both the preset and postset of t:

Case	Proof
$(p,t) \in F$	$Y'(p) = X'(p) - 1 \geq X(p) - 1 = Y(p)$
$(t,p) \in F$	$Y'(p) = X'(p) + 1 \geq X(p) + 1 = Y(p)$
$(p,t) \in R$	$Y'(p) = 0 = Y(p)$
$(t,p) \in D$	$Y'(p) = 2 \cdot X'(p) \geq 2 \cdot X'(p) = Y(p)$
$(p,t) \in Tr$	$Y'(p) = 0 = Y(p)$
$(t,p) \in Tr$	$Y'(p) = X'(p) + \sum_{(q,t) \in Tr} X'(q) \geq X(p) + \sum_{(q,t) \in Tr} X(q) = Y(p)$

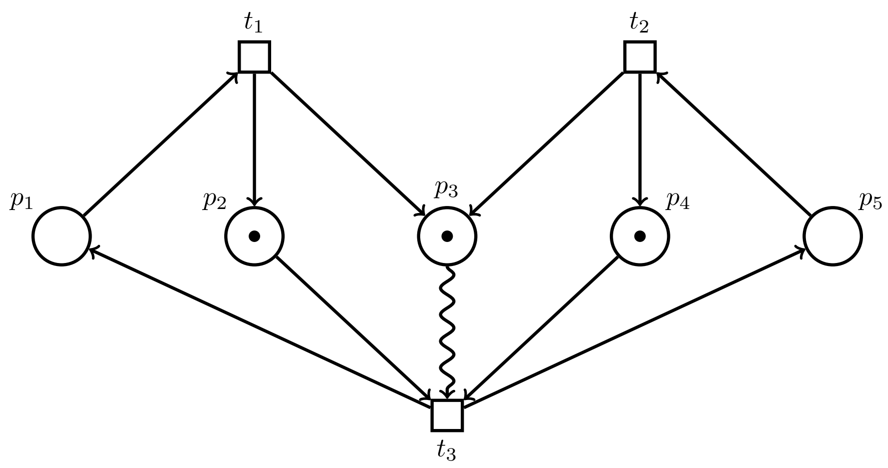
Solution 3.4

(a) Let $\mathcal{N}$ be the following Petri net with inhibitor arcs:

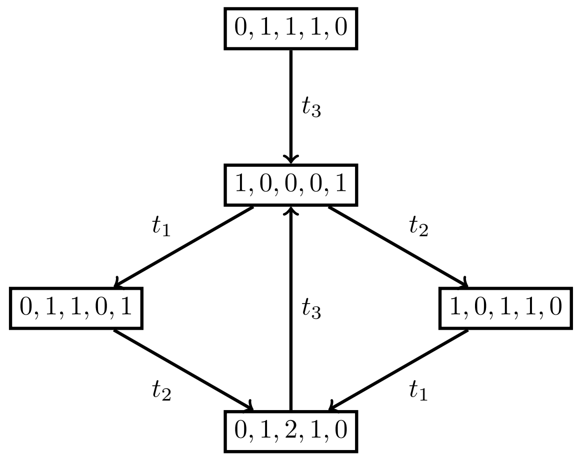


We have $(0,0) \xrightarrow{t} (0,1)$, but t is not enabled at $(1,0)$.

(b) Consider the following Petri net $\mathcal{N}$ where the arc from p_3 to t_3 is a reset arc.



It is bounded since its reachability graph is finite:



Moreover, we have $(0,1,1,1,0) \xrightarrow{\varepsilon} (0,1,1,1,0) \xrightarrow{t_3 t_1 t_2} (0,1,2,1,0)$.