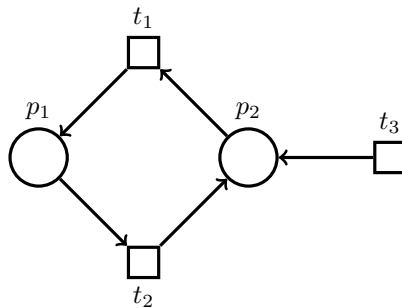


## Petri nets — Homework 3

Due 23.05.2018

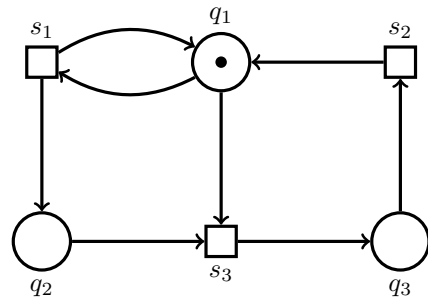
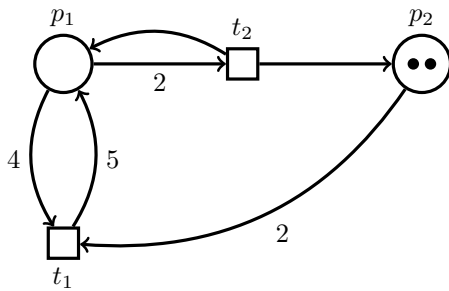
### Exercise 3.1

The algorithm COVERABILITY-GRAPH does not specify how the coverability graph should be traversed during its construction. Show that different traversal strategies can lead to different coverability graphs. More precisely, exhibit a marking  $M$  and two different coverability graphs for  $(\mathcal{N}, M)$ , where  $\mathcal{N}$  is the following net:



### Exercise 3.2

Let  $\mathcal{N}$  and  $\mathcal{N}'$  be respectively the left and right Petri nets below.



Use the backward algorithm to answer the following questions.

- Describe the set of initial markings from which  $\{p_2, p_2\}$  is coverable in  $\mathcal{N}$ . Illustrate this set.
- Determine whether  $\{q_1, q_3\}$  is coverable from  $\{q_1\}$  in  $\mathcal{N}'$ .
- Determine whether  $\{q_1, q_2\}$  is coverable from  $\{q_1\}$  in  $\mathcal{N}'$ .

**Exercise 3.3**

A net with reset, doubling and transfer arcs is a tuple  $(P, T, F, R, D, Tr)$  where  $(P, T, F)$  is a net,

$$R \subseteq P \times T, D \subseteq T \times P, Tr \subseteq (P \times T) \cup (T \times P),$$

and  $F, R, D$  and  $Tr$  are pairwise disjoint. Let  $M \in \mathbb{N}^P$  and  $t \in T$ . We say that  $t$  is enabled at  $M$  if  $M(p) > 0$  for every  $(p, t) \in F$ . Firing  $t$  at  $M$  has the following effect:

- every arc  $(p, t) \in F$  consumes a token from  $p$ ;
- every arc  $(t, p) \in F$  produces a token in  $p$ ;
- every arc  $(p, t) \in R$  empties  $p$ ;
- every arc  $(t, p) \in D$  doubles the amount of tokens in  $p$ ;
- every arc  $(p, t) \in Tr$  empties  $p$ ;
- every arc  $(t, p) \in Tr$  adds  $\sum_{(q,t) \in Tr} M(q)$  tokens to  $p$ .

Show that the backward algorithm works for this extended class of nets by showing that it is monotonic, i.e. show that for every markings  $X, X', Y \in \mathbb{N}^P$ , if  $X \rightarrow Y$  and  $X' \geq X$ , then  $X' \rightarrow Y'$  for some  $Y' \geq Y$ .

**Exercise 3.4**

- (a) Show that nets with inhibitor arcs are not monotonic.
- (b) Give a net with reset arcs  $\mathcal{N}$  and a marking  $M$  such that  $(\mathcal{N}, M)$  is bounded, but such that there exists a sequence  $M \xrightarrow{\sigma} M' \xrightarrow{\sigma'} M''$  with  $M'' \geq M'$  and  $M'' \neq M'$ .