Petri nets — Homework 2

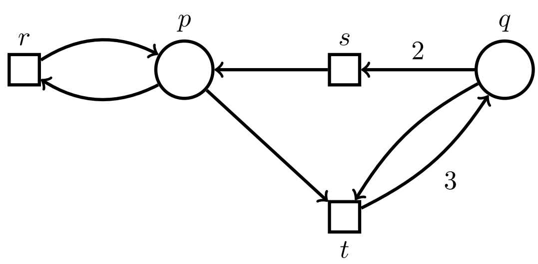
Due 08.05.2018

Exercise 2.1

(a) Give a net $\mathcal{N}$ and two markings M and M' such that $M \leq M'$, $(\mathcal{N}, M)$ is bounded, and $(\mathcal{N}, M')$ is *not* bounded.

(b) Give a net $\mathcal{N}$ and two markings M and M' such that $M \leq M'$, $(\mathcal{N}, M)$ is deadlock-free, and $(\mathcal{N}, M')$ is *not* deadlock-free.

(c) Consider the following net $\mathcal{N}$ (with weighted arcs):



Give markings M and M' such that $M \leq M'$, $(\mathcal{N}, M)$ is live, and $(\mathcal{N}, M')$ is deadlock-free but *not* live.

Exercise 2.2

Let $\mathcal{N} = (P, T, W)$ be a net with weighted arcs. Let $M, M' \in \mathbb{N}^P$, $\sigma, \sigma' \in T^*$ and $t \in T$ be such that $M \xrightarrow{\sigma t \sigma'} M'$. Prove or disprove the following statements:

(a) if t does not consume any token, i.e $W(p, t) = 0$ for every $p \in P$, then $M \xrightarrow{t \sigma \sigma'} M'$.

(b) if t consumes no more tokens than it produces, i.e $W(p, t) \leq W(t, p)$ for every $p \in P$, then $M \xrightarrow{t \sigma \sigma'} M'$.

(c) if t does not produce any token, i.e. $W(t, p) = 0$ for every $p \in P$, then $M \xrightarrow{\sigma \sigma' t} M'$.

(d) if t produces no more tokens than it consumes, i.e. $W(t, p) \leq W(p, t)$ for every $p \in P$, then $M \xrightarrow{\sigma \sigma' t} M'$.

Exercise 2.3

Show that nets with place capacities and nets with weighted arcs are equivalent to standard nets. More precisely, sketch two algorithms solving the two following problems. The worst-case running time of your algorithms should be exponential.

(a)

INPUT: a net with place capacities $\mathcal{N} = (S, T, F, K)$, and two markings M and M'.

OUTPUT: a net $\mathcal{N}' = (S', T', F')$, and two markings L and L', such that $M \xrightarrow{*} M'$ in $\mathcal{N}$ if and only if $L \xrightarrow{*} L'$ in $\mathcal{N}'$.

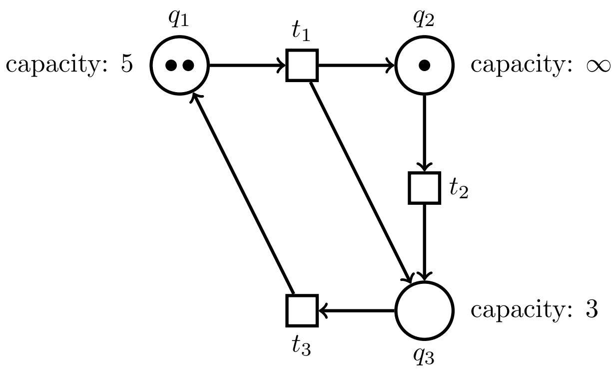
Apply your algorithm on the net below to the left with $M = \{2 \cdot q_1, q_2\}$ and $M' = \{2 \cdot q_1, q_3\}$.

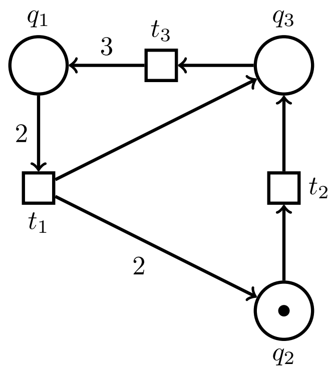
(b)

INPUT: a net with weighted arcs $\mathcal{N} = (S, T, W)$ and a markings M and M'.

OUTPUT: a net $\mathcal{N}' = (S', T', F')$, and two markings L and L', such that $M \xrightarrow{*} M'$ in $\mathcal{N}$ if and only if $L \xrightarrow{*} L'$ in $\mathcal{N}'$.

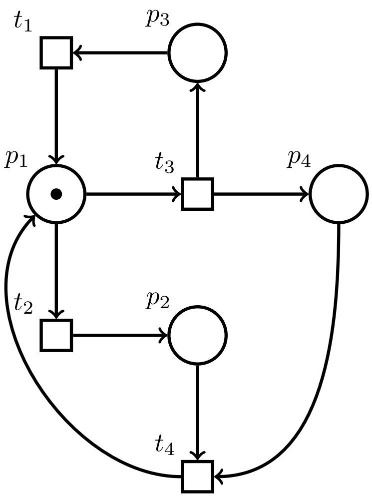
Apply your algorithm on the net below to the right with $M = \{q_2\}$ and $M' = \{q_1, 2 \cdot q_2, q_3\}$.





Exercise 2.4

Consider the following net $\mathcal{N} = (P, T, F)$:



(a) Draw a coverability graph for $(\mathcal{N}, \{p_1\})$.

(b) Is $(\mathcal{N}, \{p_1\})$ bounded? If so, why? If not, which places are bounded?

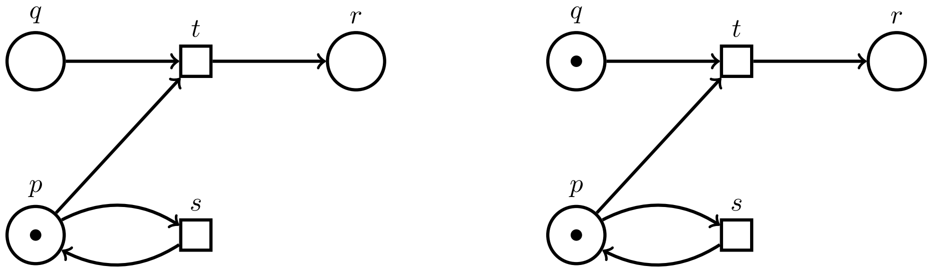
(c) Describe the set of markings coverable from $\{p_1\}$.

(d) We say that a Petri net $(\mathcal{N}, M_0)$ terminates if all its firing sequences are finite. Does $(\mathcal{N}, \{p_1\})$ terminate? Justify your answer.

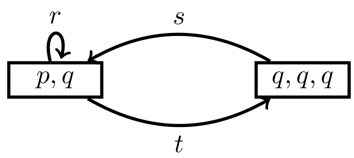
Solution 2.1

(a) The following net is bounded from the empty marking since its reachability set is empty. However, it is not bounded from $\{p\}$ since repetitively firing t increases the number of tokens in q.

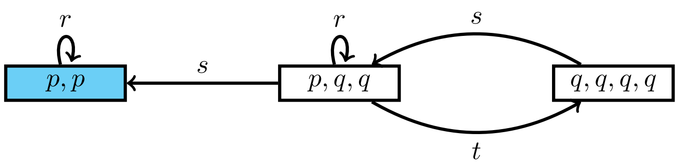
(b) The following net is deadlock-free from $\{p\}$ since s is always enabled. However, it is not deadlock-free from $\{p, q\}$ since $\{p, q\} \xrightarrow{t} \{r\}$ and $\{r\}$ is dead.



(c) $\mathcal{N}$ is live from $M = \{q, q, q\}$, since the reachability graph of $(\mathcal{N}, M)$ is strongly connected and it enables all transitions:



Let us build the reachability graph of $(\mathcal{N}, M')$ where $M' = \{q, q, q, q\}$:



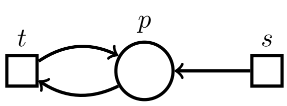
$\mathcal{N}$ is deadlock-free from M', since each marking of the reachability graph enables a transition. However, $\mathcal{N}$ is not live from M', since the bottom strongly connected component colored in blue only enables r.

Solution 2.2

(a) True. Let $A, A' \in \mathbb{N}^P$ be such that $M \xrightarrow{\sigma} A \xrightarrow{t} A' \xrightarrow{\sigma'} M'$. Since $W(p, t) = 0$ for every $p \in P$, t is enabled at any marking. In particular, $A' - A \geq \mathbf{0}$. Thus, $M \xrightarrow{t} M + (A' - A)$ and, by monotonicity, $M + (A' - A) \xrightarrow{\sigma} A + (A' - A)$. Therefore,

$$M \xrightarrow{t} M + (A' - A) \xrightarrow{\sigma} A' \xrightarrow{\sigma'} M'.$$

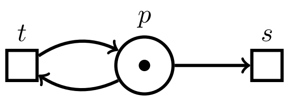
(b) False. Consider the following Petri net:



We have $0 \xrightarrow{st} 1$ and $W(p, t) = W(t, p)$, yet ts cannot be fired from 0.

(c) True. The proof is symmetric to (a).

(d) False. Consider the following Petri net:



We have $1 \xrightarrow{ts} 0$ and $W(t,p) = W(p,t)$, yet ts cannot be fired from 1.

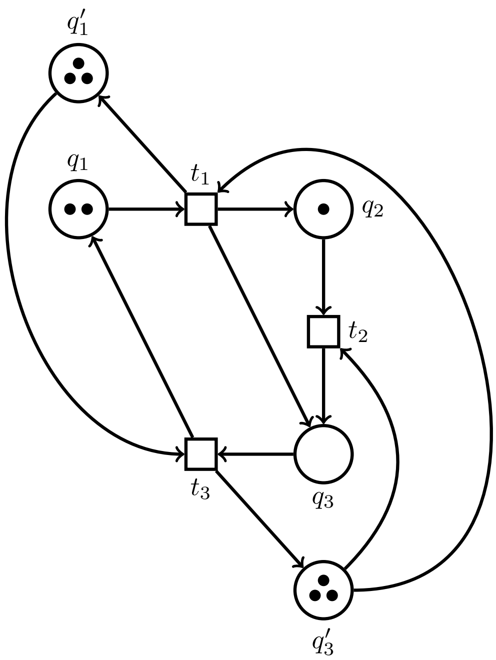
Solution 2.3

(a) We define $\mathcal{N}' = (S', T', F')$ as:

$$\begin{aligned} S' &= S \cup \{q' : q \in P \text{ s.t. } K(q) \neq \infty\}, \\ T' &= T, \\ F' &= F \cup \{(q', t) : (t, q) \in F', K(q) \neq \infty\} \cup \{(t, q') : (q, t) \in F', K(q) \neq \infty\}. \end{aligned}$$

Marking L is defined as the marking such that $L(q) = M(q)$ for every $q \in Q$ and $L(q') = K(q) - M(q)$ for every $q \in Q$ such that $K(q) \neq \infty$. Marking L' is defined similarly.

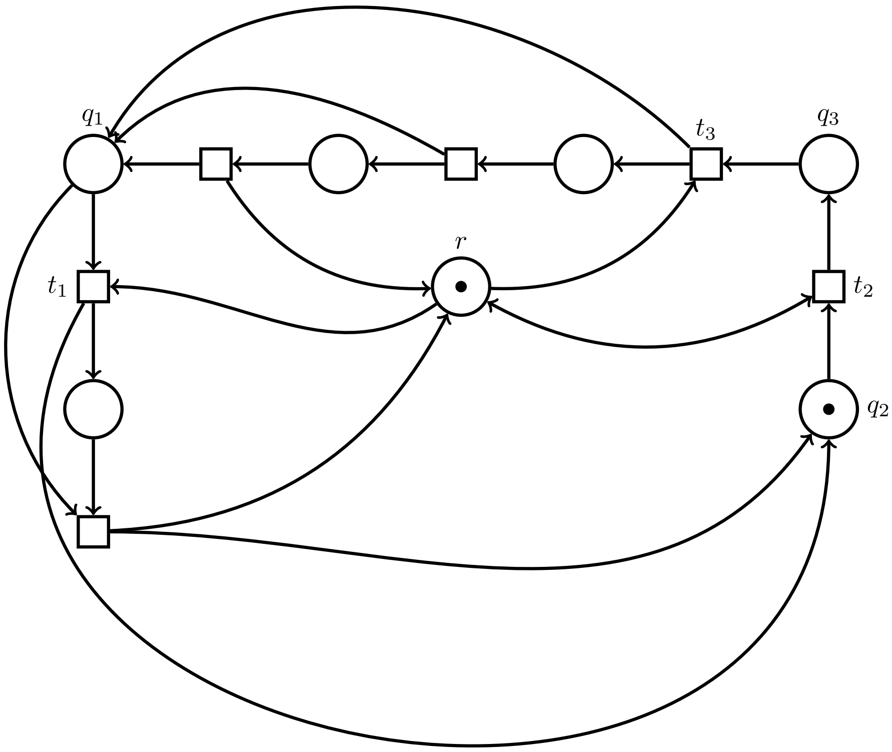
The resulting net is:



and the resulting markings are:

$$\begin{aligned} L &= \{2 \cdot q_1, 3 \cdot q_1', q_2, 3 \cdot q_3'\}, \\ L' &= \{q_1, 4 \cdot q_1', 2 \cdot q_2, q_3, 2 \cdot q_3'\}. \end{aligned}$$

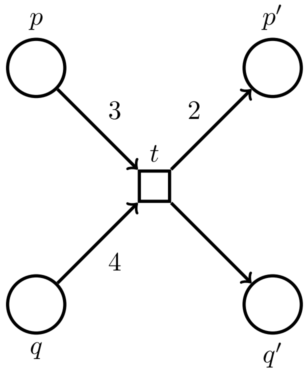
(b) Let us first give a net for the given net with weighted arcs:



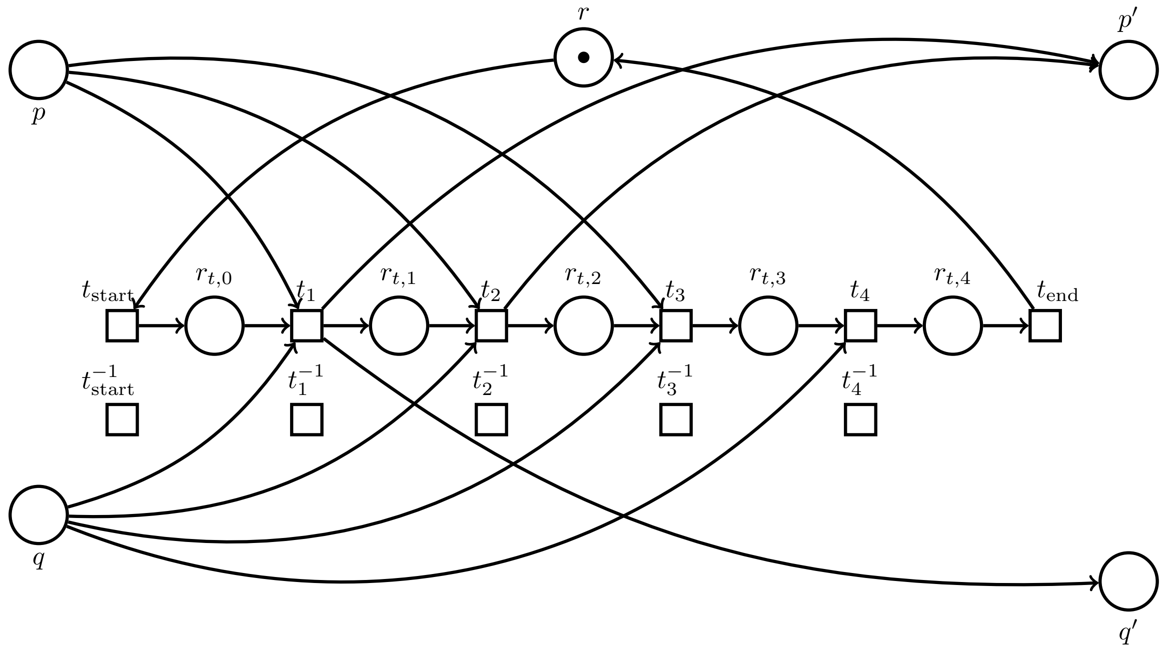
where the markings are:

$$L = \{q_2, r\},$$
$$L' = \{q_1, 2 \cdot q_2, q_3, r\}.$$

More generally, a transition such as:



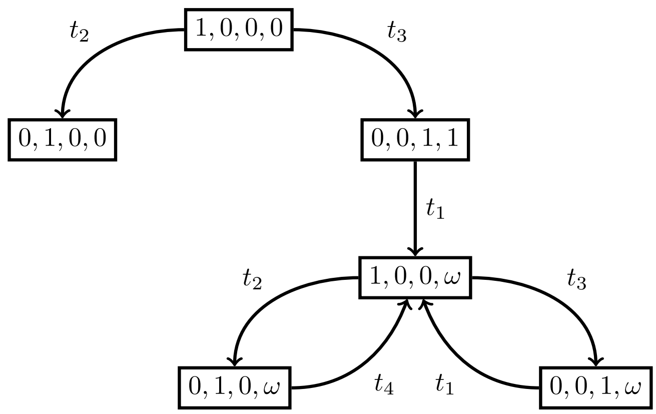
can be converted into the following gadget:



where s^{-1} denotes the inverse transition of s, i.e., $(p, s^{-1}) \in F \iff (s, p) \in F$ and $(s^{-1}, p) \in F \iff (p, s) \in F$ for every place p. Note that place r must be shared by all gadgets.

Solution 2.4

(a) The following is a coverability graph where nodes are labeled with respect to the total order $p_1 < p_2 < p_3 < p_4$:



(b) It is not bounded since some markings of the graph contain ω. Places p_1, p_2 and p_3 are bounded because no marking of the graph contains an ω in the three first components.

★ This can also be tested with `LoLA` as follows:

```
> lola pn_2-4.lola -f "AG (p1 < oo)" --search=cover
lola: result: yes
lola: The net satisfies the given formula.


> lola pn_2-4.lola -f "AG (p2 < oo)" --search=cover
lola: result: yes
lola: The net satisfies the given formula.


> lola pn_2-4.lola -f "AG (p3 < oo)" --search=cover
lola: result: yes
```

```
lola: The net satisfies the given formula.
```

```
> lola pn_2-4.lola -f "AG (p4 < oo)" --search=cover
lola: result: no
lola: The net does not satisfy the given formula.
```

(c) $\{M \in \mathbb{N}^P : M(p_1) + M(p_2) + M(p_3) = 1\}$.

(d) No, it does not terminate. A Petri net terminates if and only if its coverability graph contains a cycle or at least one node with an ω. The above coverability graph contains both. In particular, $t_3 t_1 (t_2 t_4)^\omega$ is an infinite firing sequence.