Technische Universität München | Summer term 2018
I7
Prof. J. Esparza / Dr. M. Blondin | 02.05.2018

Petri nets — Homework 2

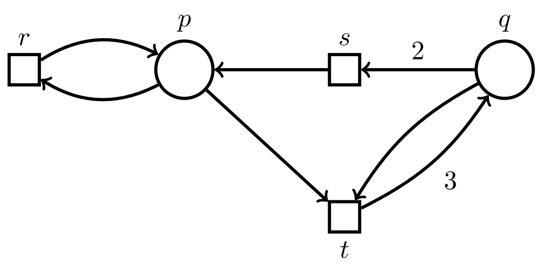
Due 08.05.2018

Exercise 2.1

(a) Give a net $\mathcal{N}$ and two markings M and M' such that $M \leq M'$, $(\mathcal{N}, M)$ is bounded, and $(\mathcal{N}, M')$ is *not* bounded.

(b) Give a net $\mathcal{N}$ and two markings M and M' such that $M \leq M'$, $(\mathcal{N}, M)$ is deadlock-free, and $(\mathcal{N}, M')$ is *not* deadlock-free.

(c) Consider the following net $\mathcal{N}$ (with weighted arcs):



Give markings M and M' such that $M \leq M'$, $(\mathcal{N}, M)$ is live, and $(\mathcal{N}, M')$ is deadlock-free but *not* live.

Exercise 2.2

Let $\mathcal{N} = (P, T, W)$ be a net with weighted arcs. Let $M, M' \in \mathbb{N}^P$, $\sigma, \sigma' \in T^*$ and $t \in T$ be such that $M \xrightarrow{\sigma t \sigma'} M'$. Prove or disprove the following statements:

(a) if t does not consume any token, i.e $W(p, t) = 0$ for every $p \in P$, then $M \xrightarrow{t \sigma \sigma'} M'$.

(b) if t consumes no more tokens than it produces, i.e $W(p, t) \leq W(t, p)$ for every $p \in P$, then $M \xrightarrow{t \sigma \sigma'} M'$.

(c) if t does not produce any token, i.e. $W(t, p) = 0$ for every $p \in P$, then $M \xrightarrow{\sigma \sigma' t} M'$.

(d) if t produces no more tokens than it consumes, i.e. $W(t, p) \leq W(p, t)$ for every $p \in P$, then $M \xrightarrow{\sigma \sigma' t} M'$.

Exercise 2.3
Show that nets with place capacities and nets with weighted arcs are equivalent to standard nets. More precisely, sketch two algorithms solving the two following problems. The worst-case running time of your algorithms should be exponential.

(a)

INPUT: a net with place capacities $\mathcal{N} = (S, T, F, K)$, and two markings M and M'.

OUTPUT: a net $\mathcal{N}' = (S', T', F')$, and two markings L and L', such that $M \xrightarrow{*} M'$ in $\mathcal{N}$ if and only if $L \xrightarrow{*} L'$ in $\mathcal{N}'$.

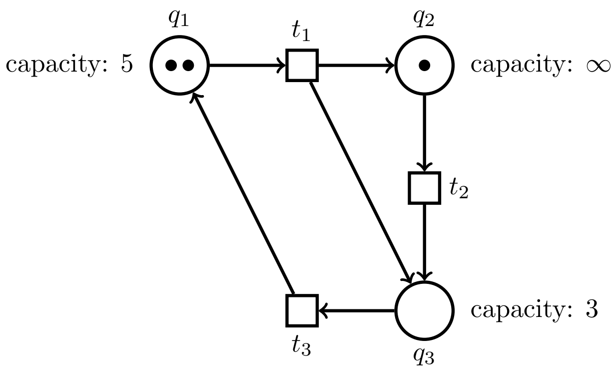
Apply your algorithm on the net below to the left with $M = \{2 \cdot q_1, q_2\}$ and $M' = \{2 \cdot q_1, q_3\}$.

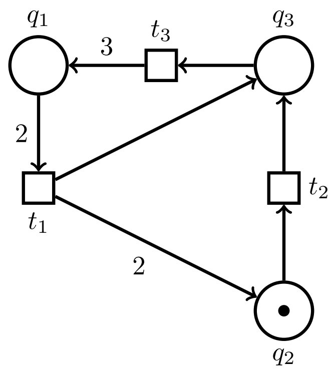
(b)

INPUT: a net with weighted arcs $\mathcal{N} = (S, T, W)$ and a markings M and M'.

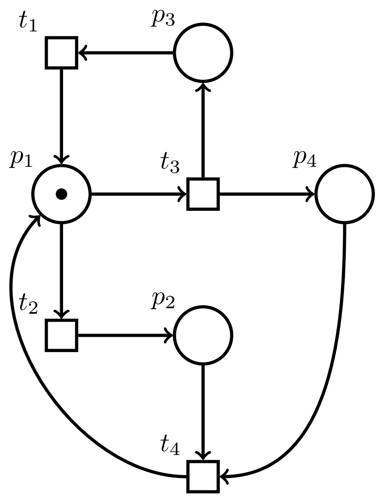
OUTPUT: a net $\mathcal{N}' = (S', T', F')$, and two markings L and L', such that $M \xrightarrow{*} M'$ in $\mathcal{N}$ if and only if $L \xrightarrow{*} L'$ in $\mathcal{N}'$.

Apply your algorithm on the net below to the right with $M = \{q_2\}$ and $M' = \{q_1, 2 \cdot q_2, q_3\}$.





Exercise 2.4
Consider the following net $\mathcal{N} = (P, T, F)$:



(a) Draw a coverability graph for $(\mathcal{N}, \{p_1\})$.

(b) Is $(\mathcal{N}, \{p_1\})$ bounded? If so, why? If not, which places are bounded?

(c) Describe the set of markings coverable from $\{p_1\}$.

(d) We say that a Petri net $(\mathcal{N}, M_0)$ terminates if all its firing sequences are finite. Does $(\mathcal{N}, \{p_1\})$ terminate? Justify your answer.