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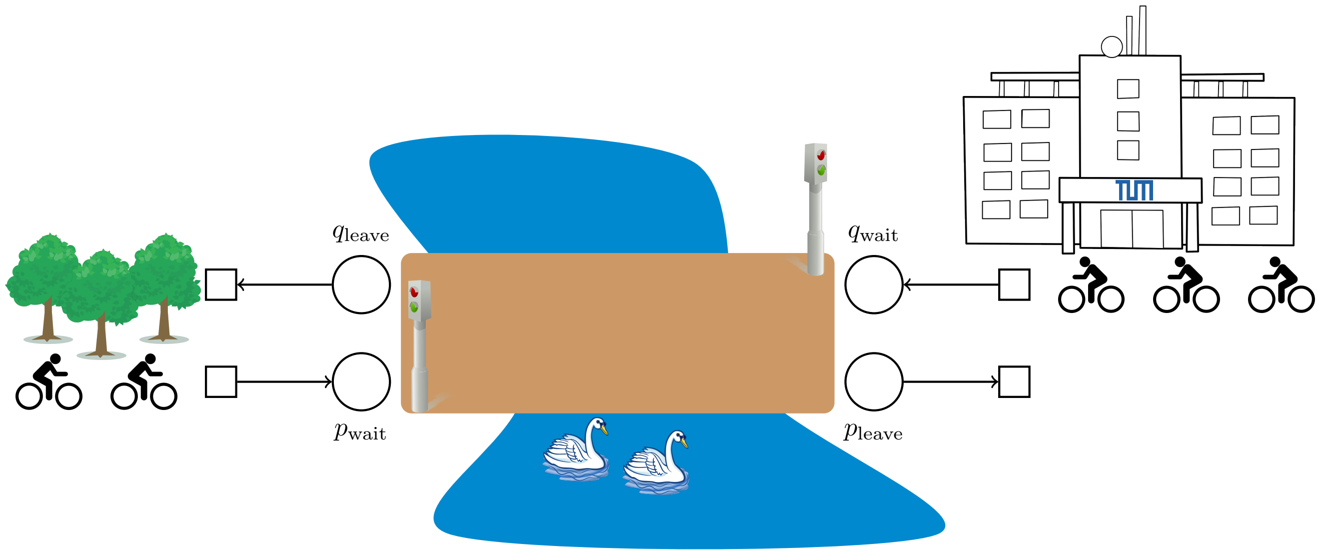
Summer term 2018
10.04.2018

Petri nets — Exercise sheet 1

Due 18.04.2018

Exercise 1.1 (adapted from [1, ex. 2.22])

Consider a new (fictive) bridge connecting TUM to the other side of the Isar. Since this bridge is narrow, it can only be used in one direction at a time. Moreover, for safety reasons, there should not be more than six cyclists at a time on the bridge. The university wants the bridge to be equipped with a system controlling green and red lights on both ends of the bridge. For each direction, when the green light is on, cyclists are allowed to get onto the bridge; and when the red light is on, cyclists are *not* allowed to get onto the bridge.



Model the bridge as a Petri net (with weighted arcs) by extending the partial model shown above. Cyclists should flow from p_{wait} to p_{leave}, and from q_{wait} to q_{leave}. Assume that, initially, the left green light is on, the right red light is on, and the bridge is empty. Make sure that the model respects safety, i.e. that bikes are not allowed to go in opposite directions simultaneously, and that the bridge cannot hold more than six bikes.

Exercise 1.2

Consider Lamport's 1-bit mutual exclusion algorithm:

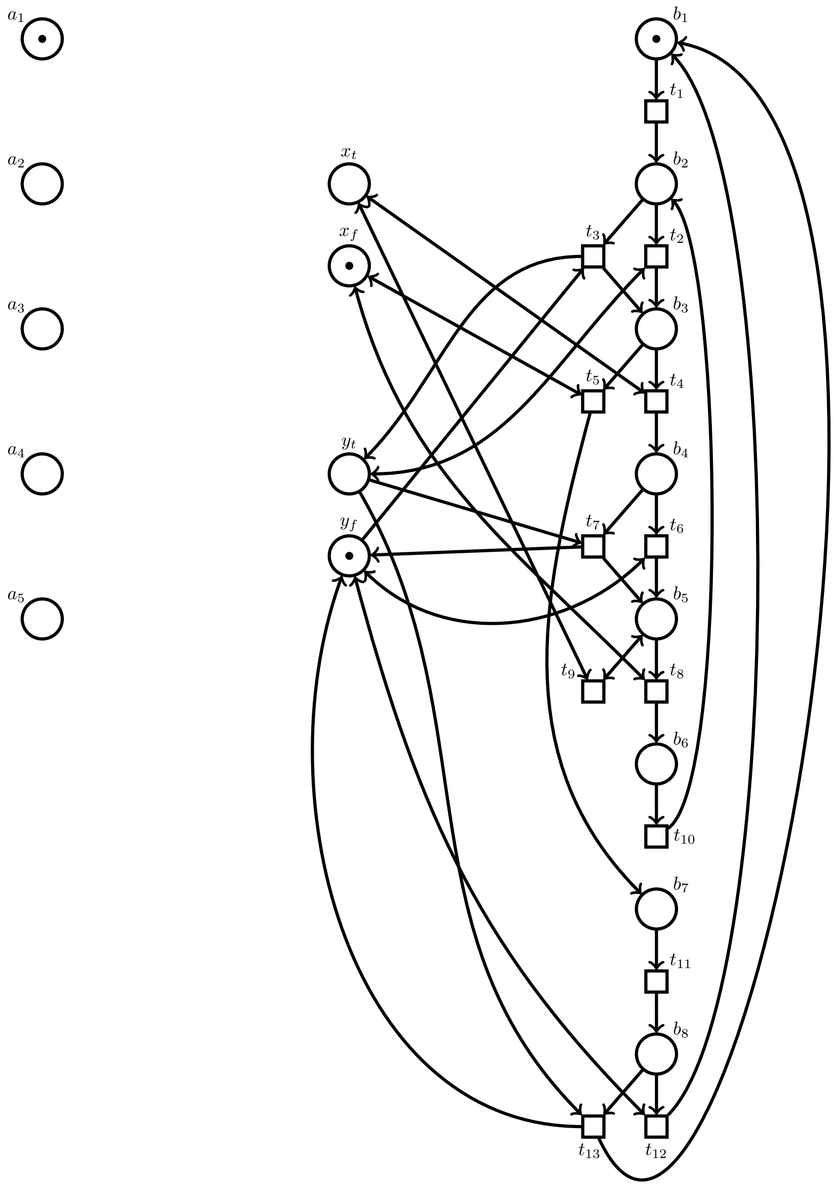
First process

```
1.  while True:
2.    x = True
3.    while y: pass
4.    # critical section
5.    x = False
```

Second process

```
1.  while True:
2.    y = True
3.    if x then:
4.      y = False
5.      while x: pass
6.      goto 2
7.    # critical section
8.    y = False
```

The algorithm can be modeled by a Petri net $\mathcal{N}$ where each program location (i.e. line of code of a process) is associated to a place, and where the shared binary variables `x` and `y` are associated to two places each. In more details, $\mathcal{N} = (P, T, F)$ where $P = \{a_1, \ldots, a_5, b_1, \ldots, b_8, x_t, x_f, y_t, y_f\}$. A token in a_i (resp. b_i) indicates that the first (resp. second) process is at line i; a token in x_t (resp. y_t) indicates that `x` (resp. `y`) has value `True`; and a token in x_f (resp. y_f) indicates that `x` (resp. `y`) has value `False`. The initial marking of $\mathcal{N}$ is $M_0 = \{a_1, b_1, x_f, y_f\}$. We give a partial Petri net that only models the second process:



(a) Complete the above Petri net $\mathcal{N}$ so that it also models the first process. You should not add new places, only transitions and arcs. Note that `pass` is a "no operation", i.e. an operation without any effect.

(b) Complete the given `APT` file for $\mathcal{N}$ accordingly, and verify whether

(i) $(\mathcal{N}, M_0)$ is bounded;

(ii) $(\mathcal{N}, M_0)$ is live.

(c) Complete the given `LoLA` file for $\mathcal{N}$ accordingly, and verify whether

(i) $(\mathcal{N}, M_0)$ is deadlock-free;

(ii) a process can be at multiple program locations at the same time;

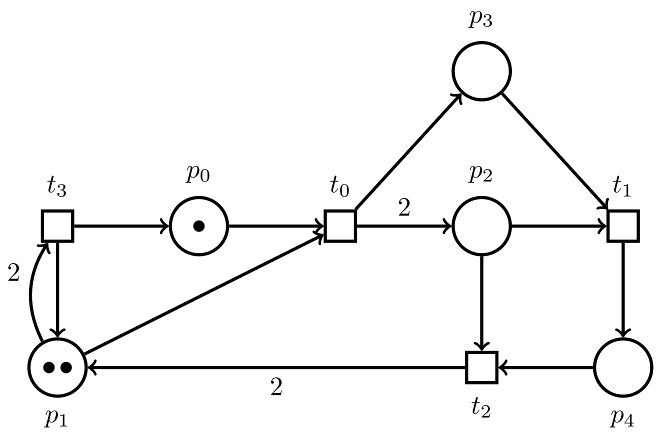
(iii) whether both processes can reach their critical sections simultaneously.

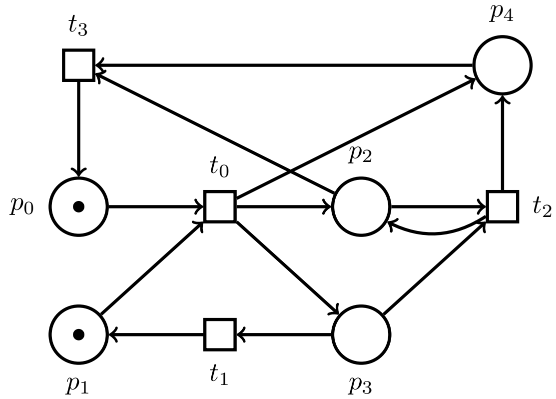
Exercise 1.3
For each Petri net $(\mathcal{N}, M_0)$ below:

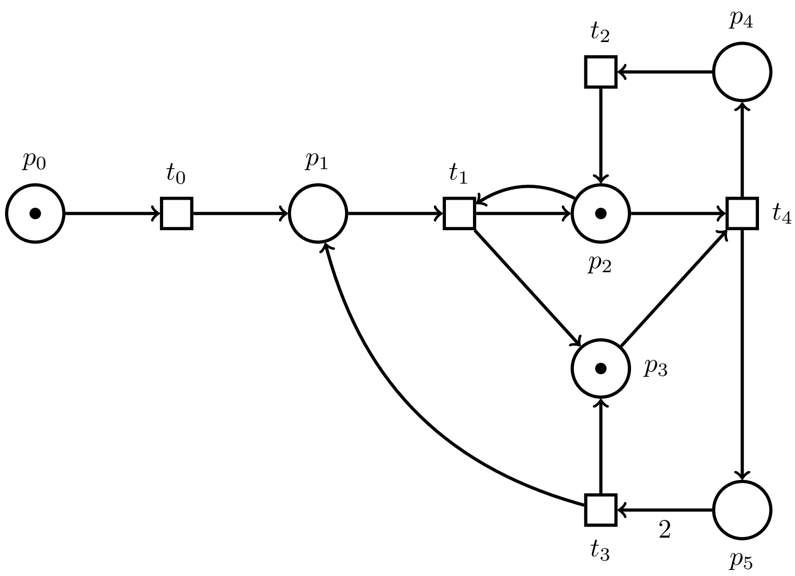
(a) construct the reachability graph of $(\mathcal{N}, M_0)$.

(b) say whether $(\mathcal{N}, M_0)$ is bounded, deadlock-free and/or live. If it is bounded, give the smallest k such that it is k-bounded. Justify your answers.

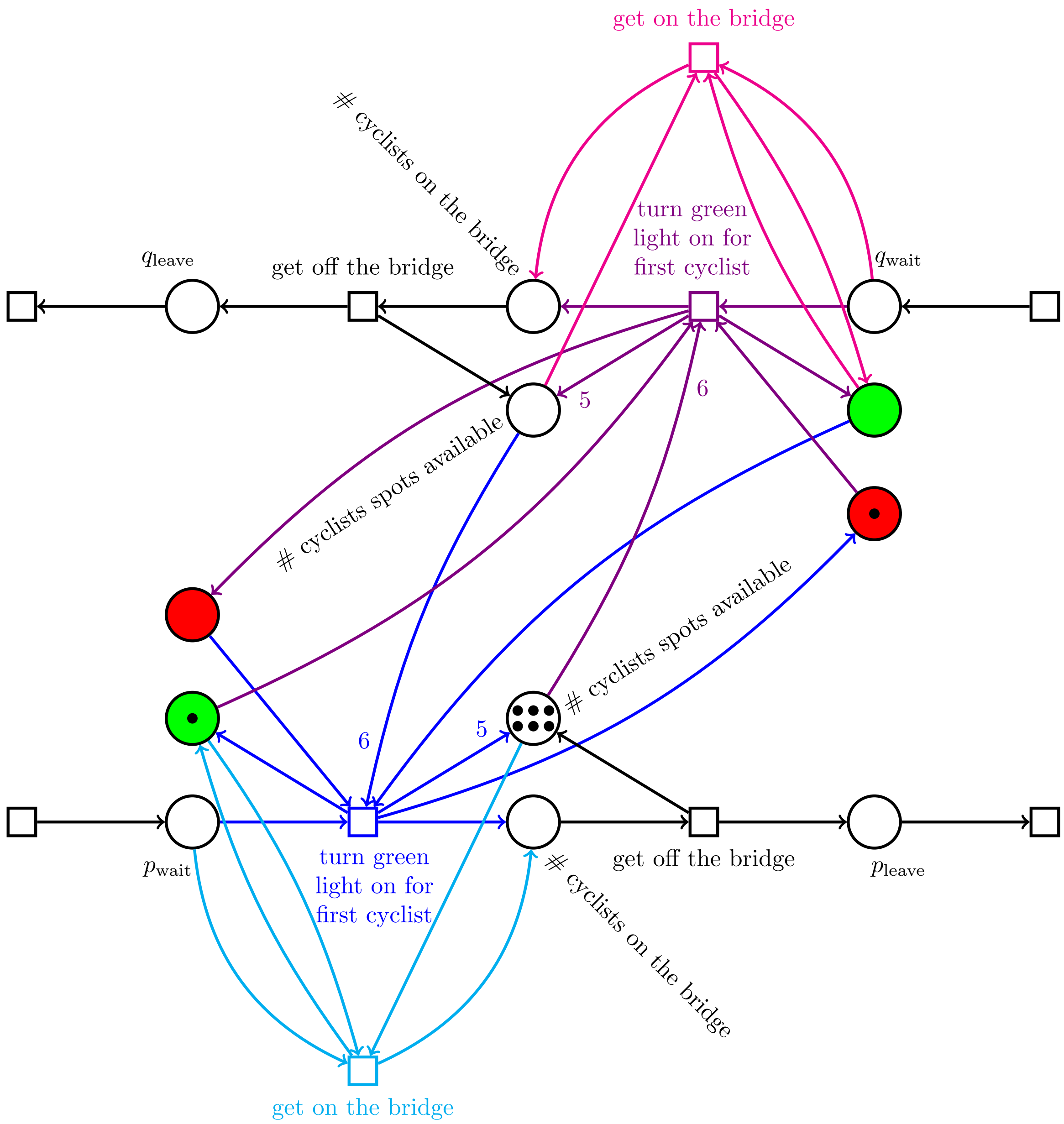
(c) give the subnet $\mathcal{N}' = (P', T', F')$ of $\mathcal{N}$ such that $P' = \{p_0, p_1, p_2, p_4\}$ and $|T'|$ is maximal.







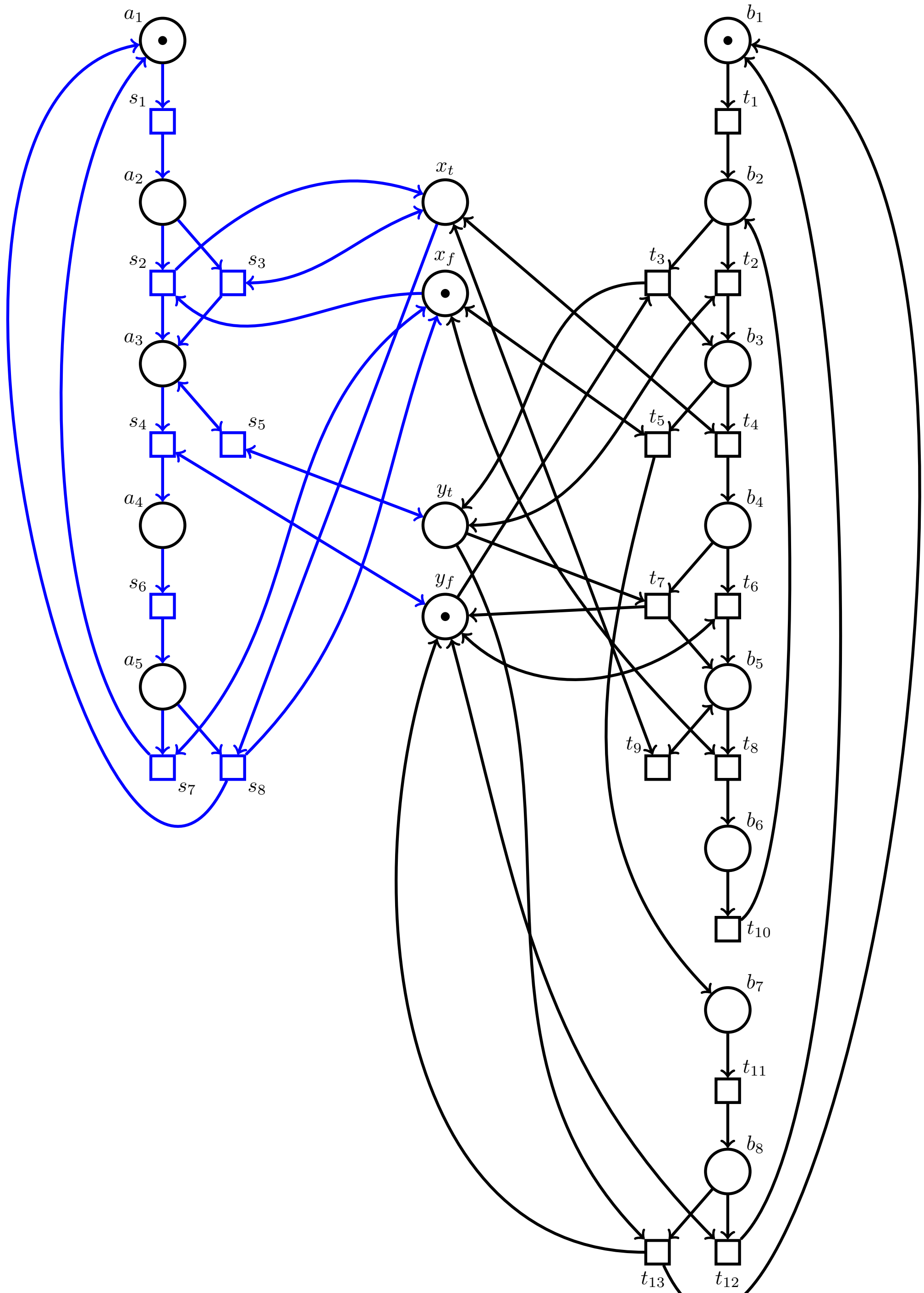
Solution 1.1 (adapted from [1, ex. 2.22])
The bridge can be modeled as follows:



★ Note that in this modeling, the light may remain green in one direction even though six cyclists are on the bridge. This satisfies the given specification, but probably not what we would hope for. Another solution will be uploaded later.

Solution 1.2

(a)

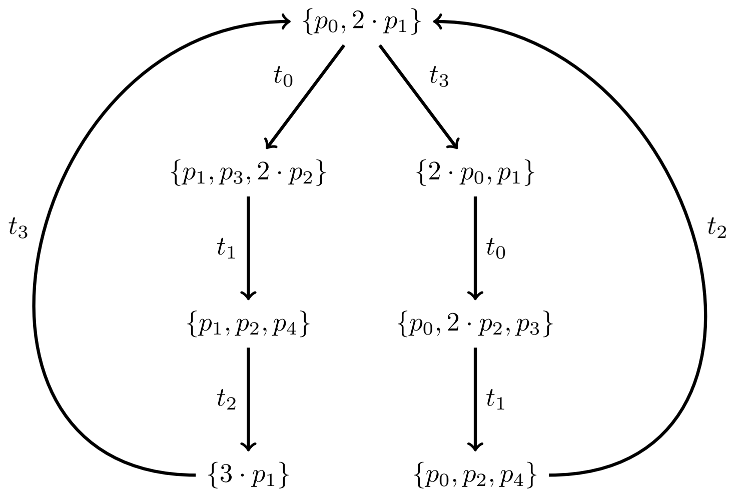


(b) (i)
```
> java -jar apt.jar bounded lamport.apt
bounded:  Yes
smallest_K: 1
```
(ii)
```
> java -jar apt.jar strongly_live lamport.apt
strongly_live:  No
```

(c) (i)
```
> lola lamport.lola -f "REACHABLE DEADLOCK"
lola:  result:  no
lola:  The net does not satisfy the given formula.
```
(ii)
```
> lola lamport.lola -f "REACHABLE (a1 + a2 + a3 + a4 + a5 > 1) OR (b1 + b2 + b3 + b4
+ b5 + b6 + b7 + b8 > 1)"
lola:  result:  no
lola:  The net does not satisfy the given formula.
```
(iii)
```
> lola lamport.lola -f "REACHABLE (a4 > 0 AND b7 > 0)"
lola:  result:  no
lola:  The net does not satisfy the given formula.
```

Solution 1.3

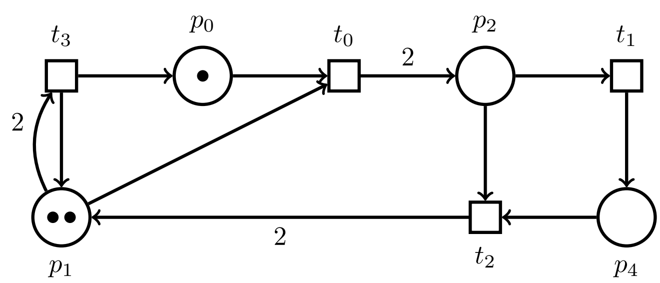
1. (a)



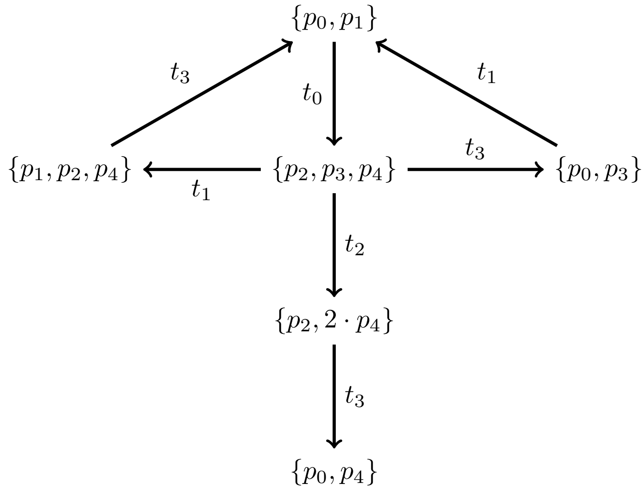
(b) It is 3-*bounded* since all markings of the reachability graph have at most three tokens in each place. It is *deadlock-free* since every marking of the reachability graph has an outgoing arc. It is *live* because for every transition t, every marking M of the reachability graph leads to a marking M' with an outgoing arc labeled by t.

★ Alternatively, liveness follows from the fact that the reachability graph is strongly connected and has an occurrence of every transition.

(c)

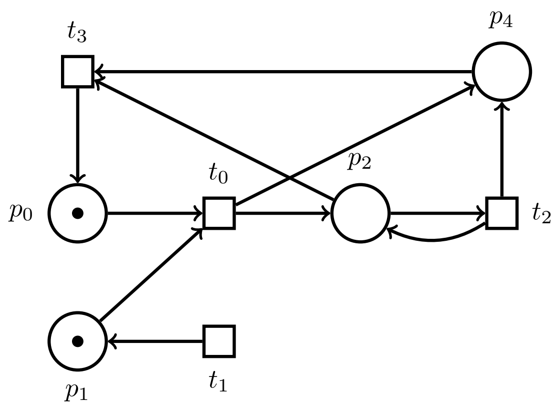


2. (a)

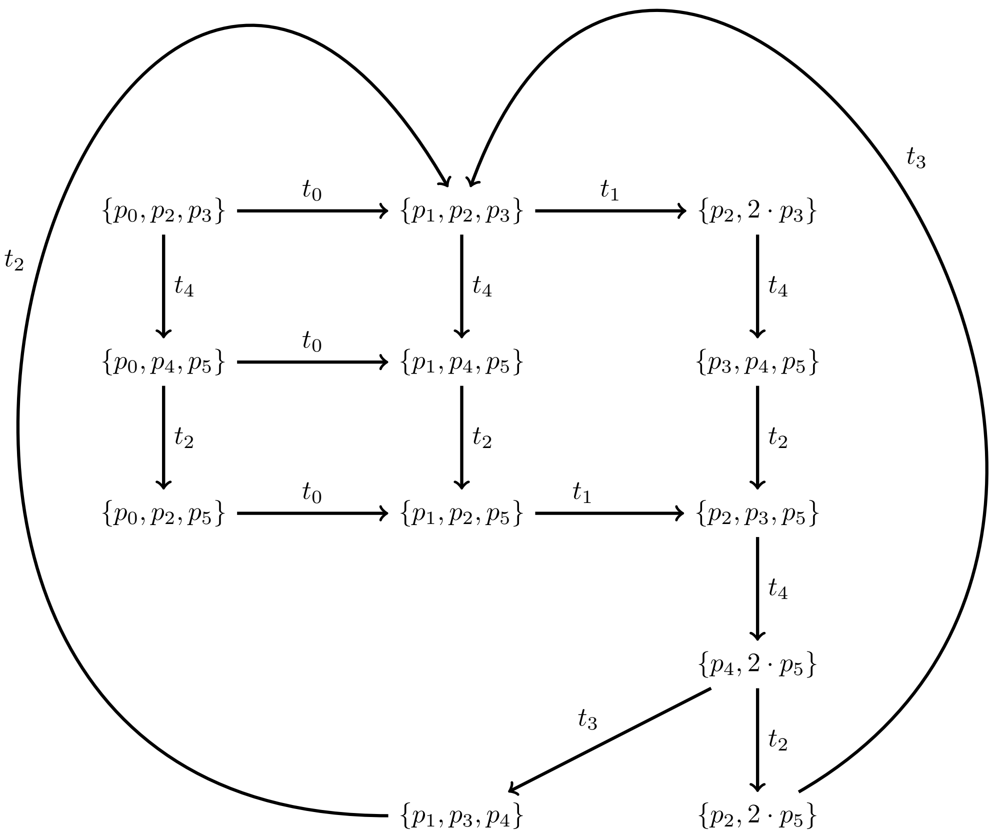


(b) It is 2-*bounded* since all markings of the reachability graph have at most two tokens in each place. It is *not deadlock-free* since $\{p_0, p_4\}$ has no successor. It is *not live* since it is not deadlock-free.

(c)



3. (a)



(b) It is *not live* since in the reachability graph has no path from $\{p_1, p_2, p_3\}$ that contains t_0. It is 2-*bounded* since all markings of the reachability graph have at most 2 tokens in each place. It is *deadlock-free* since every marking of the reachability graph has an outgoing arc.

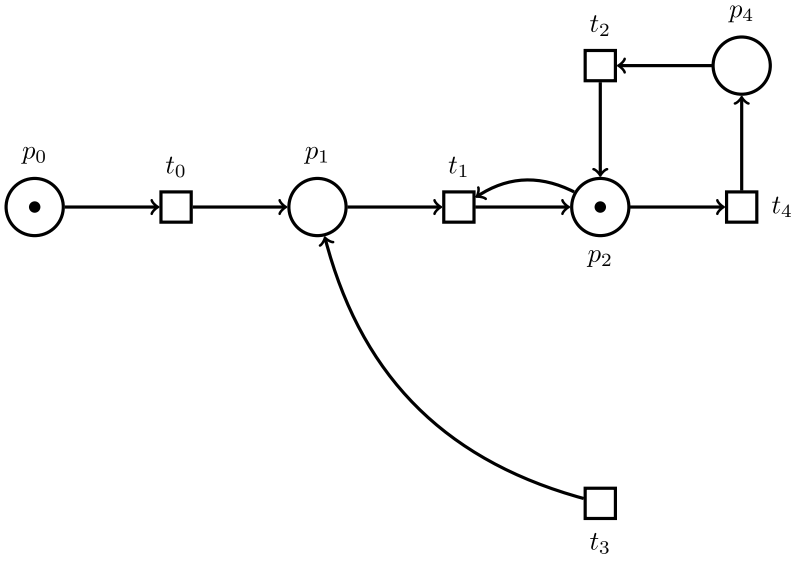
★ It is possible to show that the net is not live without inspecting the reachability graph. Note that $M_0 \xrightarrow{t_0} \{p_1, p_2, p_3\}$. Moreover, $\mathcal{N}$ has no transition that produces a token in p_0. Therefore, $\{p_1, p_2, p_3\}$ cannot reach any marking from which t_0 is enabled.

★ There is an alternative way to prove 2-boundness and deadlock-freedom without inspecting the reachability graph. Let $Q = \{p_0, p_1, p_3, p_5\}$ and $R = \{p_2, p_4\}$. We claim that $M(Q) = 2$ and $M(R) = 1$ for every reachable marking M. The claim clearly holds for M_0. Moreover, every transition of $\mathcal{N}$ consumes and produces the same amount of tokens from both Q and R, which proves the claim. Now, for the sake of contradiction, assume there exists a deadlock, e.g. there exists some reachable marking M from which no transition is enabled. By definition of transitions t_0, t_2 and t_3, this implies that

$$M(p_0) = 0,$$
$$M(p_4) = 0,$$
$$M(p_5) \leq 1,$$

and hence, by the claim, that $M(p_1) + M(p_3) \geq 1$ and $M(p_2) = 1$. In particular $M(p_1) > 0$ or $M(p_3) > 0$. If the former holds, then t_1 is enabled, if the latter holds, then t_4 is enabled. Both cases yield contradictions.

(c)



References

[1] Wil van der Aalst, Massimiliano de Leoni, Boudewijn van Dongen, and Christian Stahl. Course business information systems: exercises, 2015. Available at `http://wwwis.win.tue.nl/~wvdaalst/old/courses/BIScourse/exercise-bundle-BIS-2015.pdf`.