An SMT-Based Approach to Coverability Analysis

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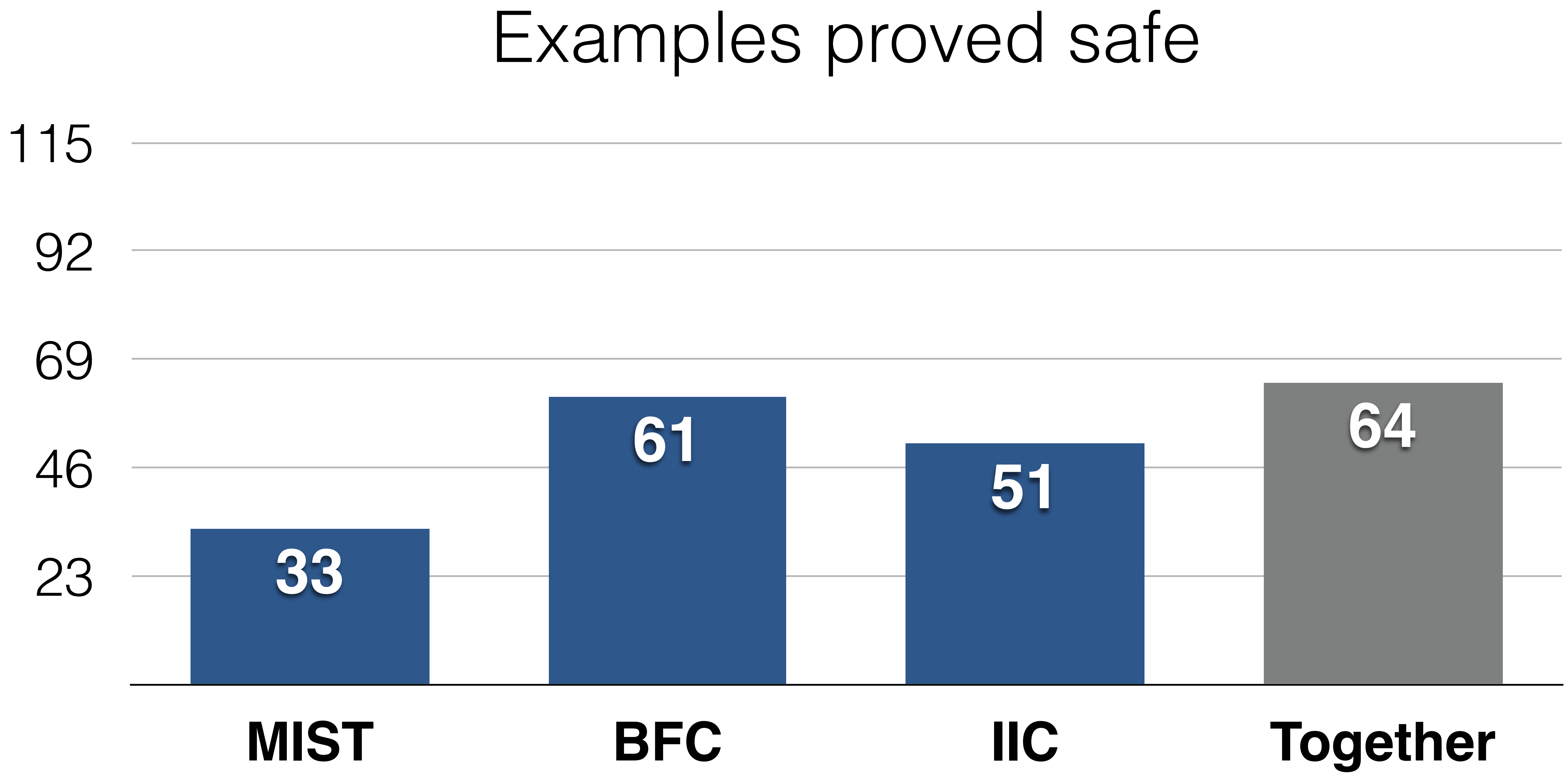
Petri net coverability is important, but difficult

- Many verification problems reduce to Petri net coverability problem
- Petri net coverability is **EXPSPACE-complete**
- Sophisticated tools and algorithms:

  MIST — Expand-enlarge-check [GRB '06]
  BFC — Minimal uncoverability proof [KKW '12]
  IIC — Incremental, inductive coverability [KMNP '13]

MIST, BFC and IIC don't scale well

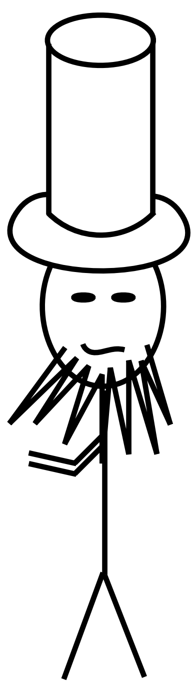


Reducing coverability to feasibility of linear constraints

Method **LinCon**:

- Based on **marking equation** [Murata '77]
  Incomplete

- Strengthened with **traps** [EM '00]
  Traps — essentially **Boolean constraints**
  Still incomplete

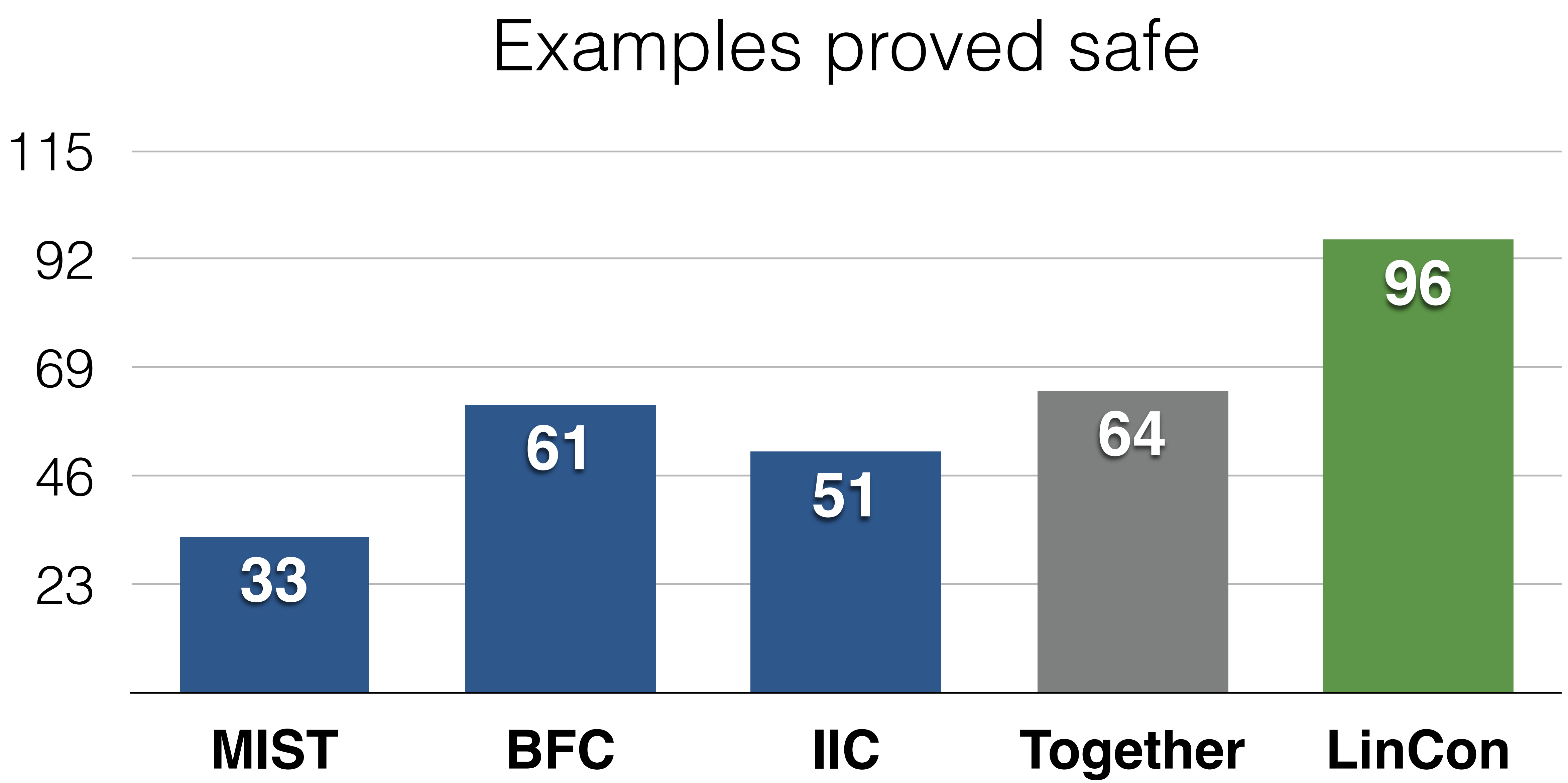


Use **SMT** for linear and Boolean constraints.

But **LinCon is incomplete.**

Does it make sense to use it?

Yes! For the right class of examples, LinCon is "quite complete"



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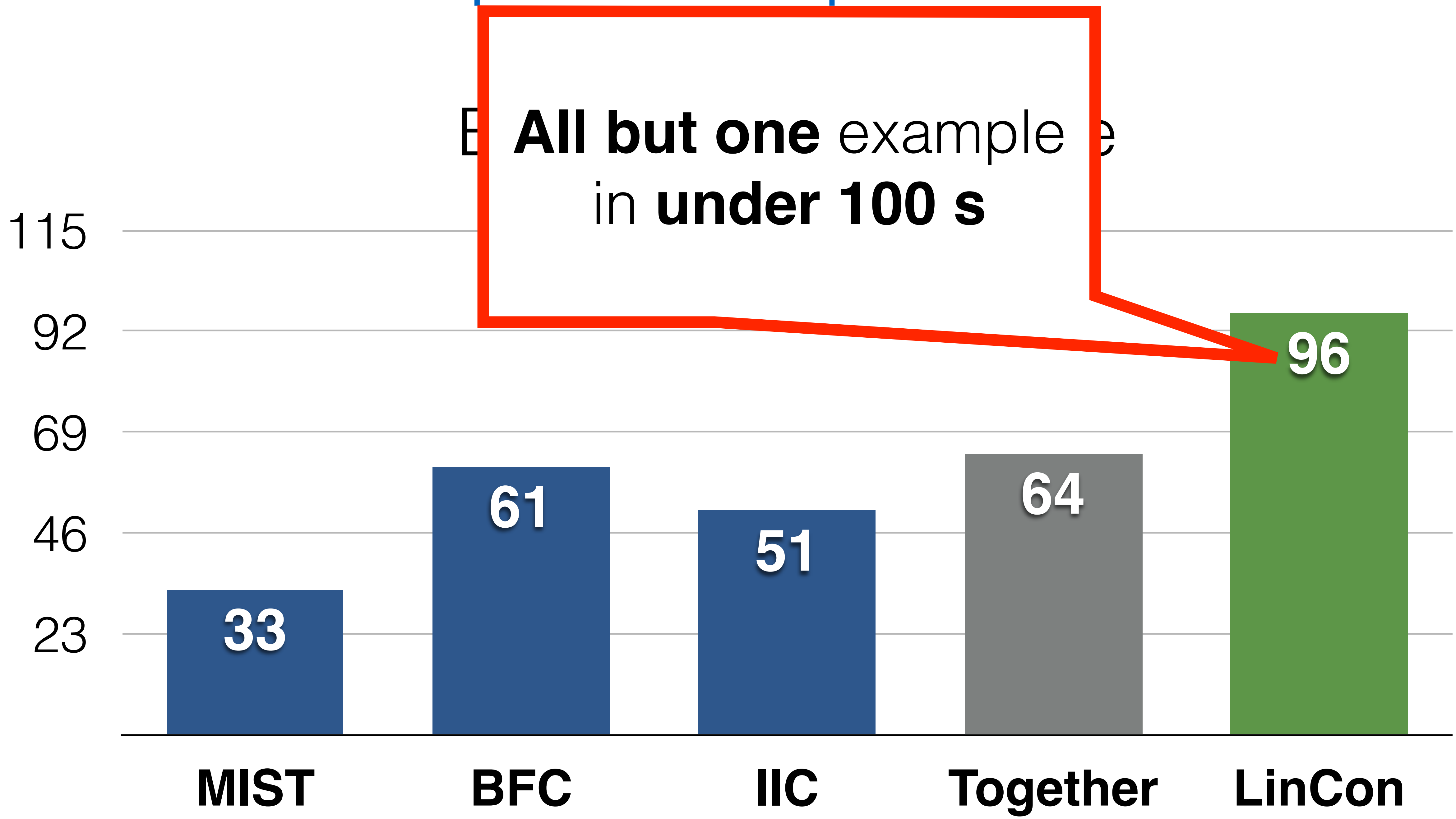

Contributions

Main contribution:

- Extensive **experimental evaluation** showing that **LinCon works well**

Also:

- Using **duality** of linear programming to derive succinct **inductive invariants**

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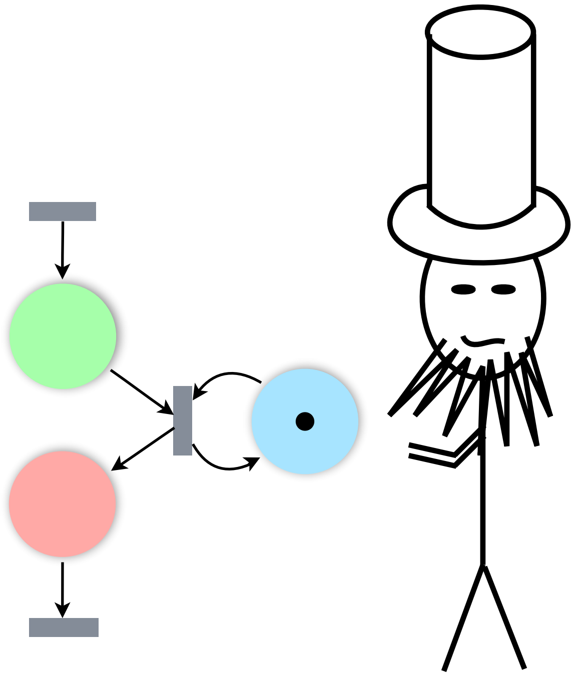
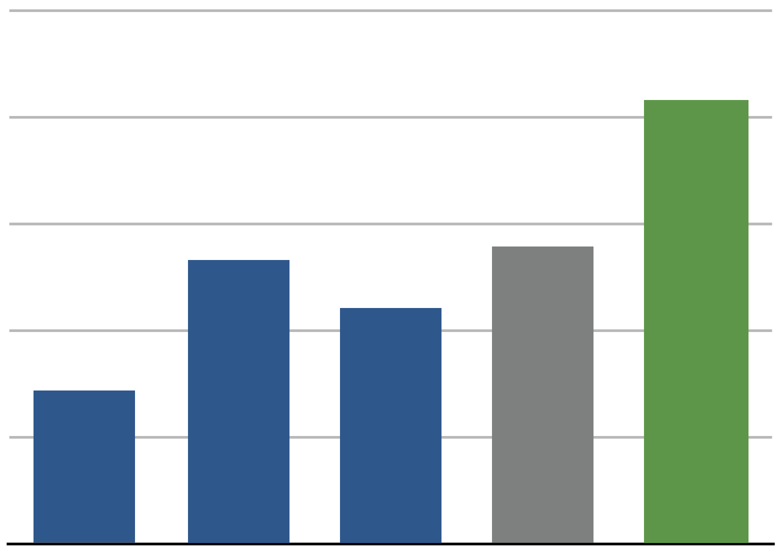
- Using **duality** of linear programming to derive succinct **inductive invariants**

In this talk

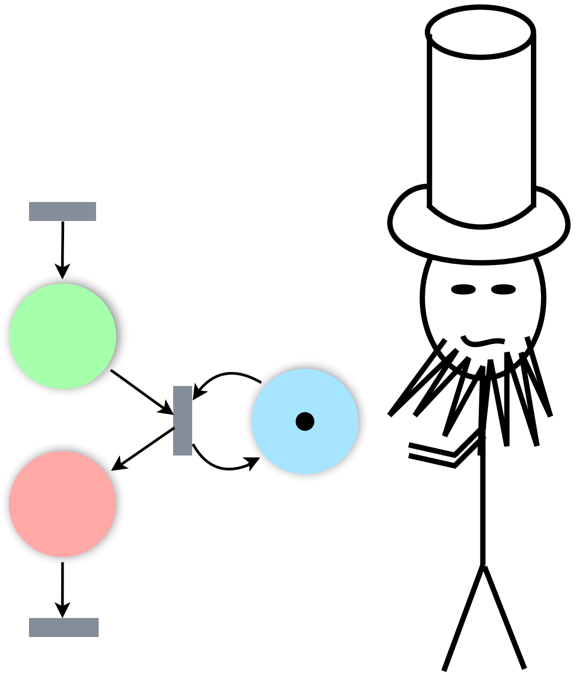


Petri nets
and LinCon



Experiments

In this talk



Petri nets
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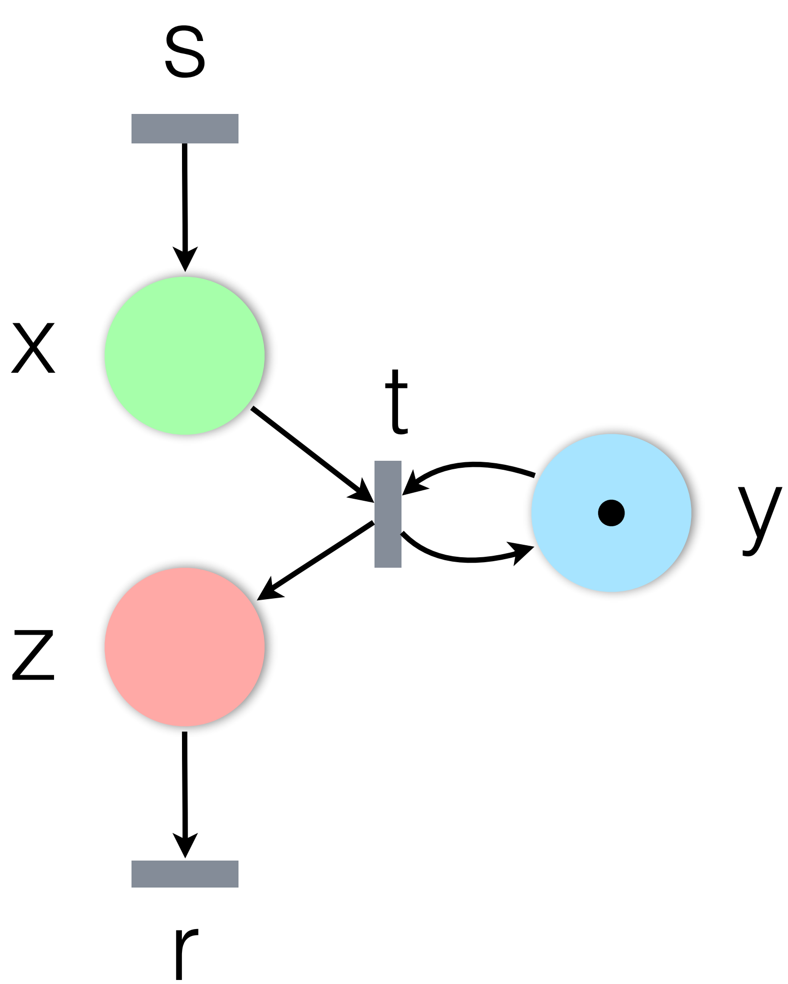
Experiments

Petri nets are state transition systems



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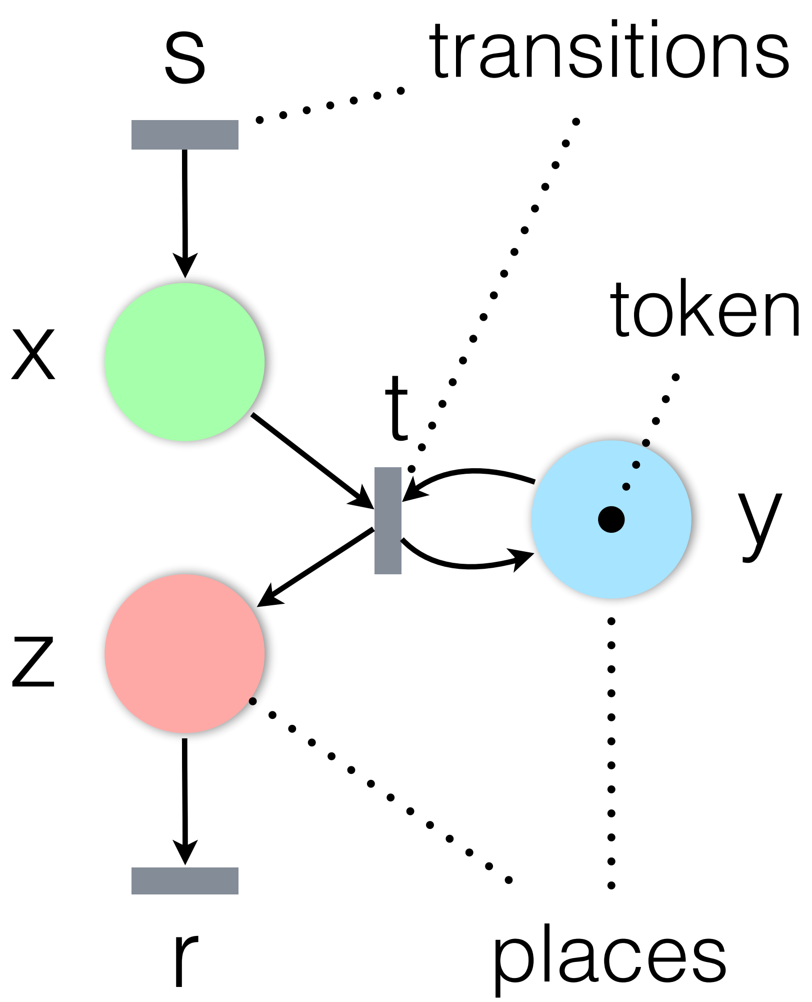

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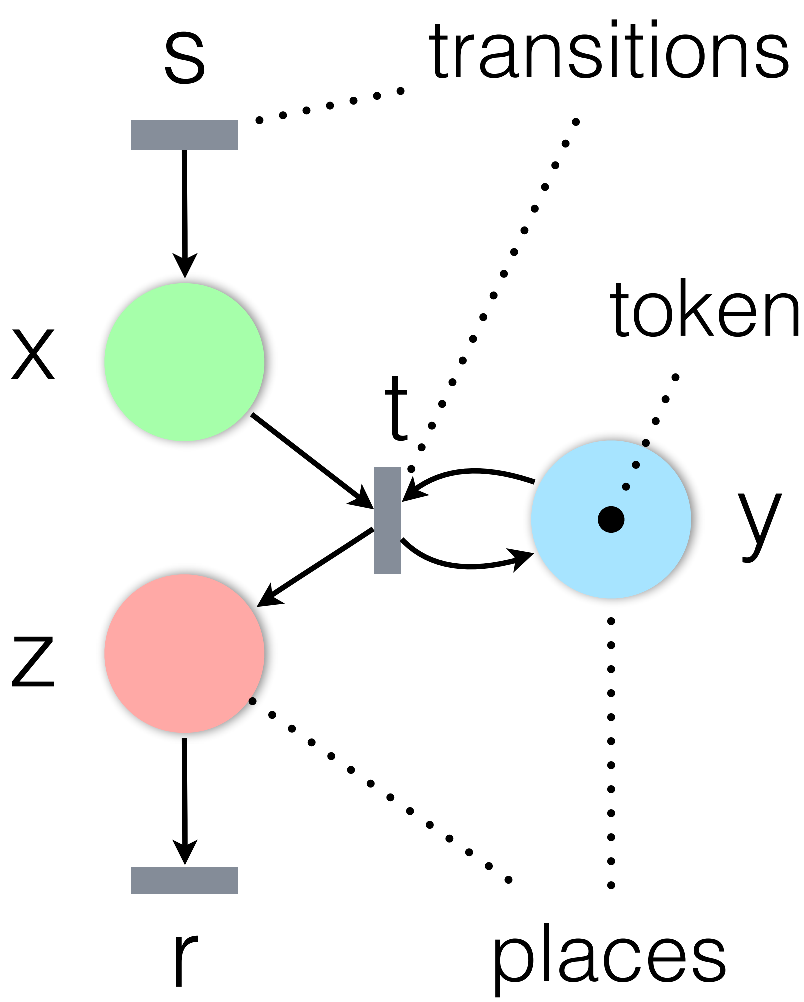
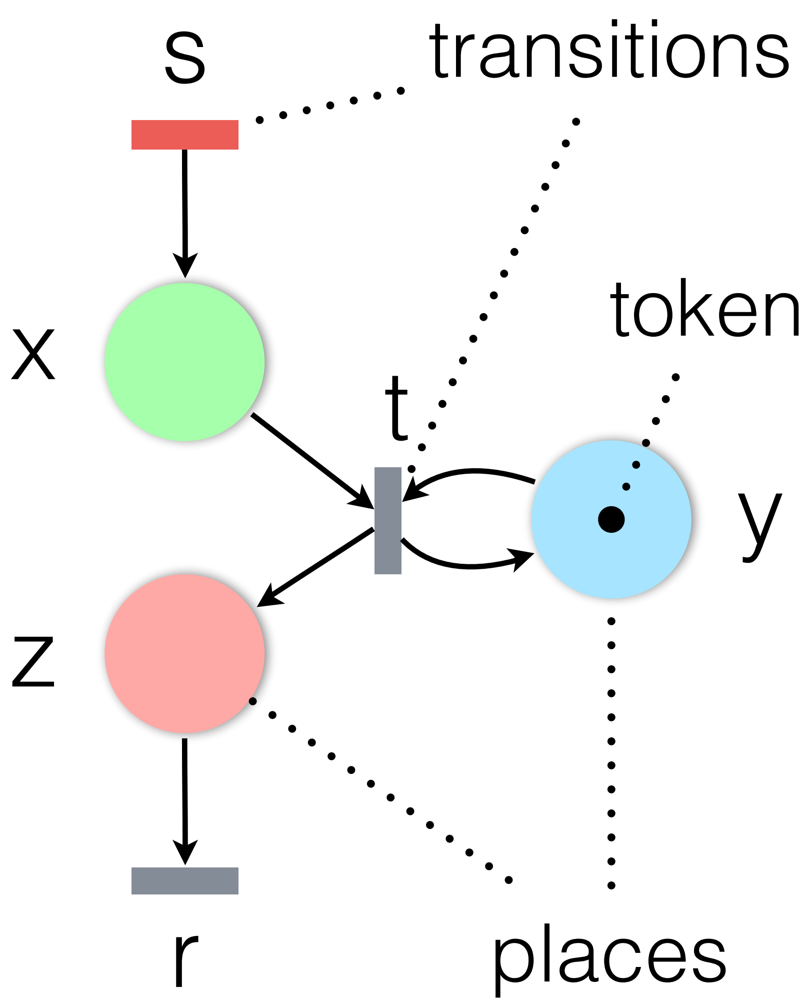


initial marking

(0, 1, 0)

Petri nets are state transition systems



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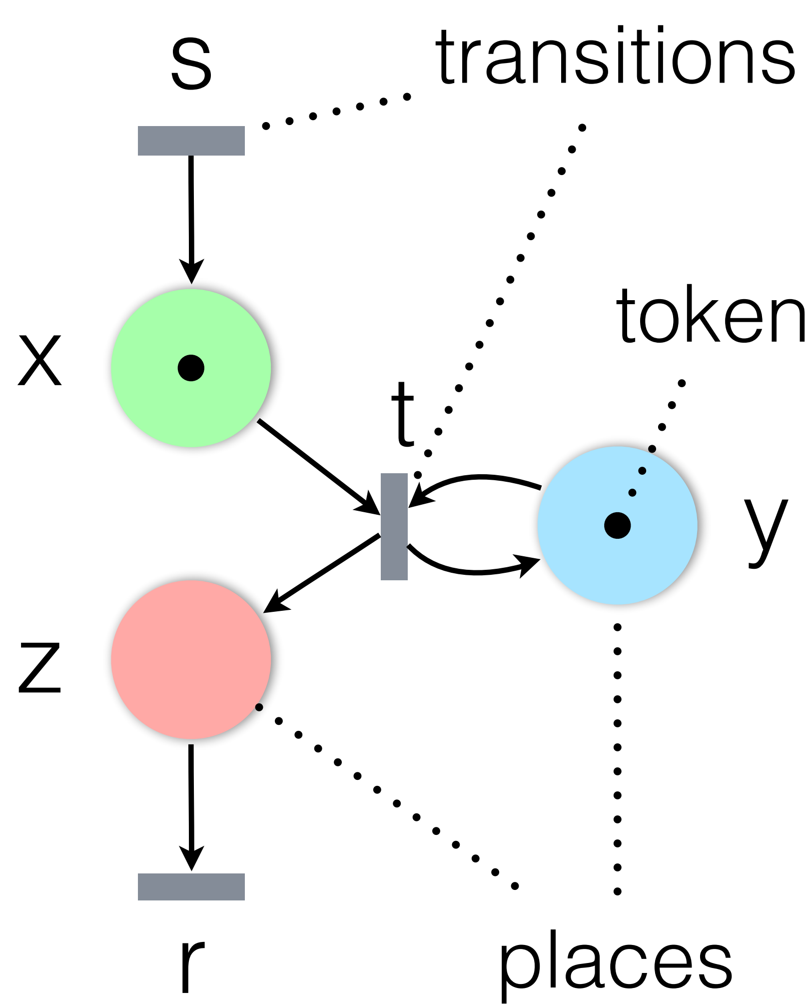
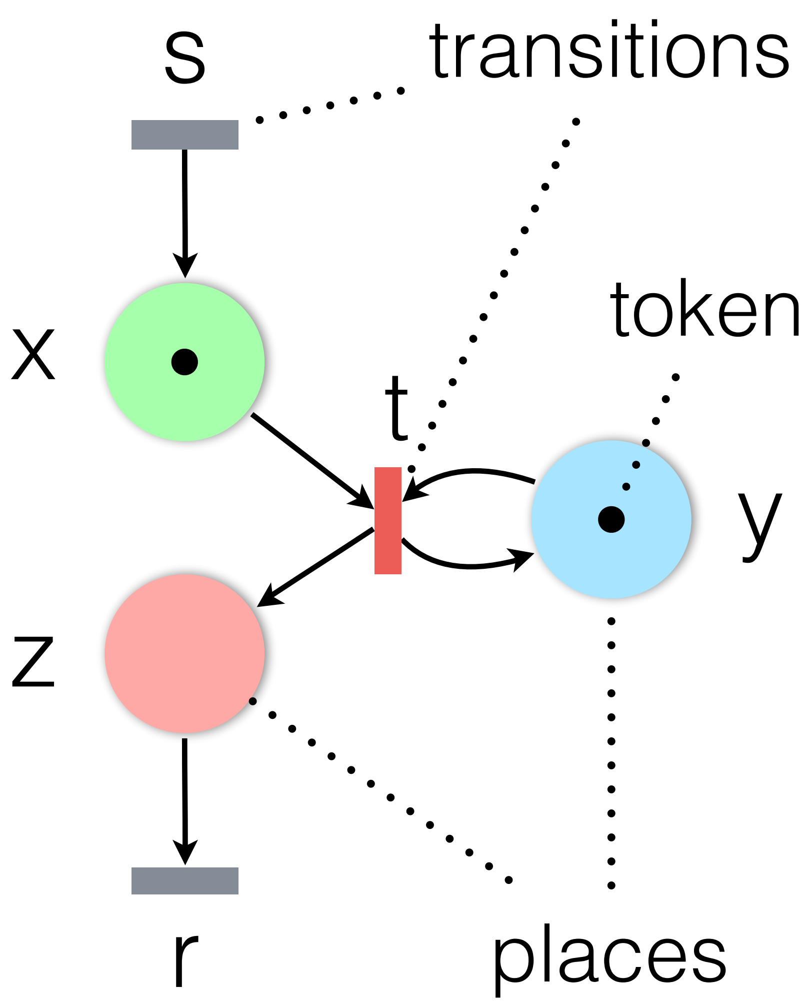


initial marking

$$\begin{array}{l} (0, 1, 0) \\ \quad\downarrow \cdots\cdots +(1, 0, 0) \\ (1, 1, 0) \end{array}$$

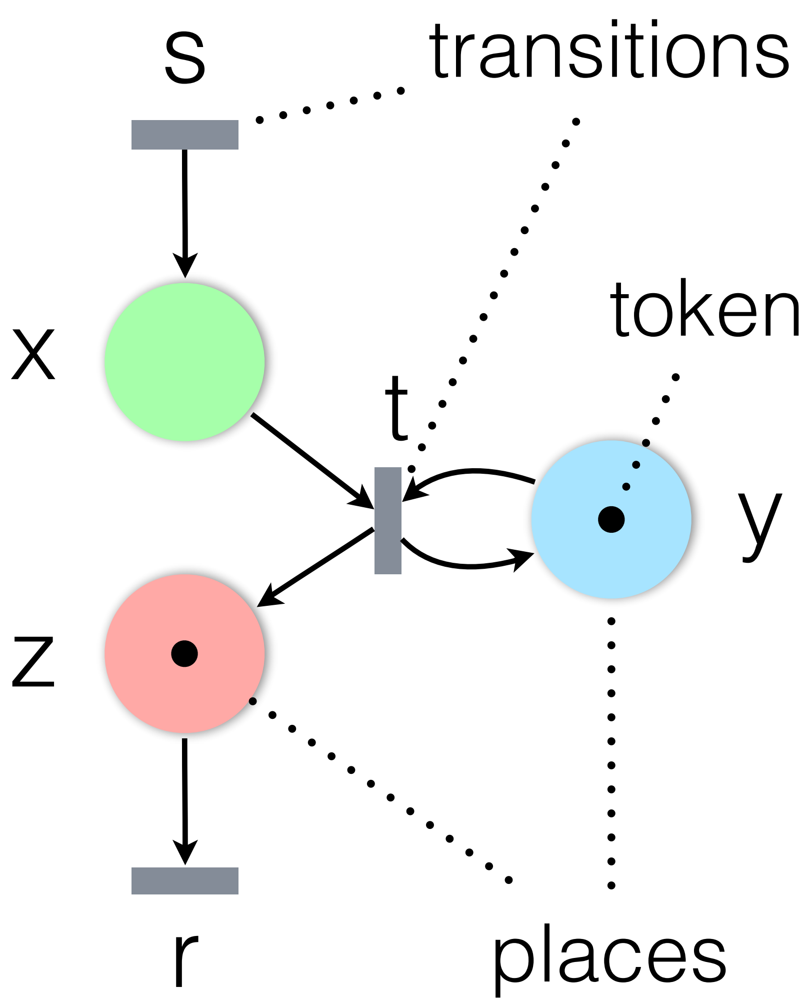
Petri nets are state transition systems



initial marking

$$(0, 1, 0) \xrightarrow{+(1, 0, 0)} (1, 1, 0)$$

Petri nets are state transition systems



initial marking

$$(0, 1, 0)$$

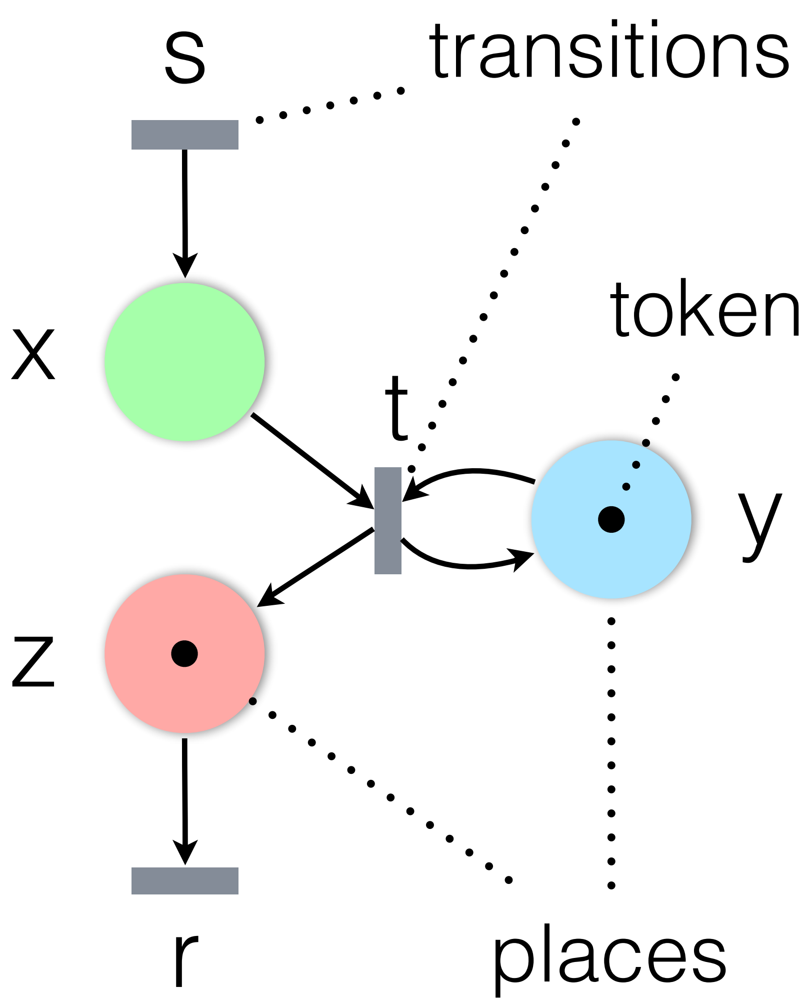
$$\big\downarrow \cdots\cdots +(1, 0, 0)$$

$$(1, 1, 0)$$

$$\big\downarrow \cdots\cdots +(-1, 0, 1)$$

$$(0, 1, 1)$$

Petri nets are state transition systems



initial marking

$(0, 1, 0)$

$\downarrow \; +(1, 0, 0)$

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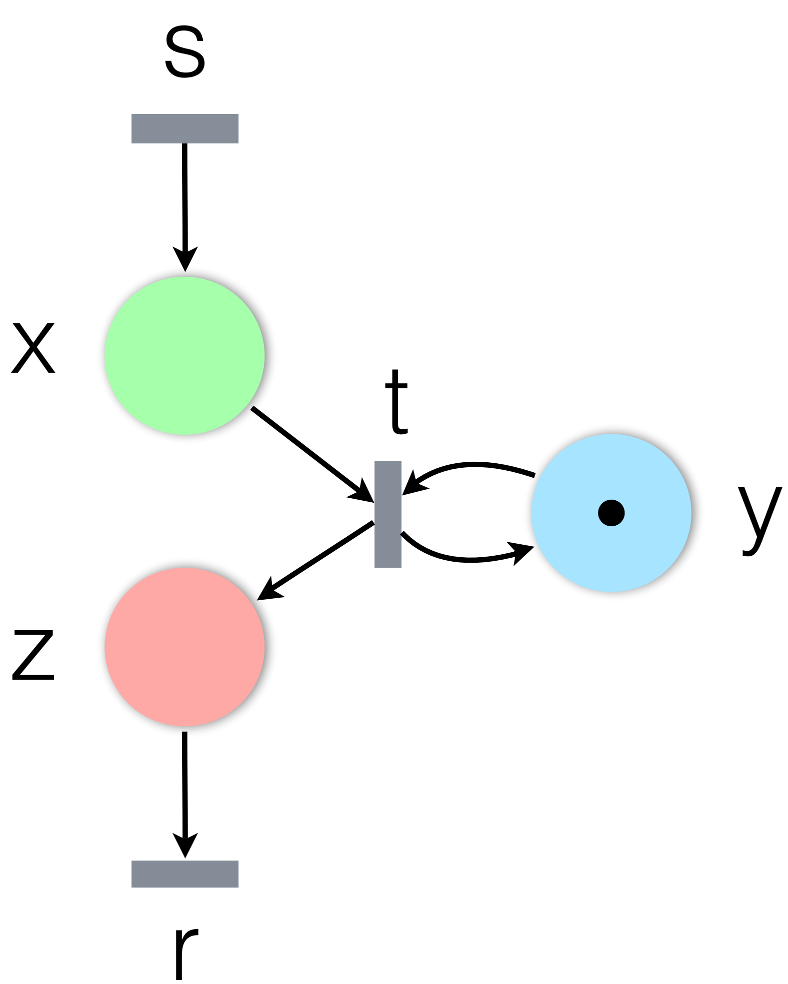
$(0, 1, 1)$

reachable markings

Reachable markings satisfy marking equation

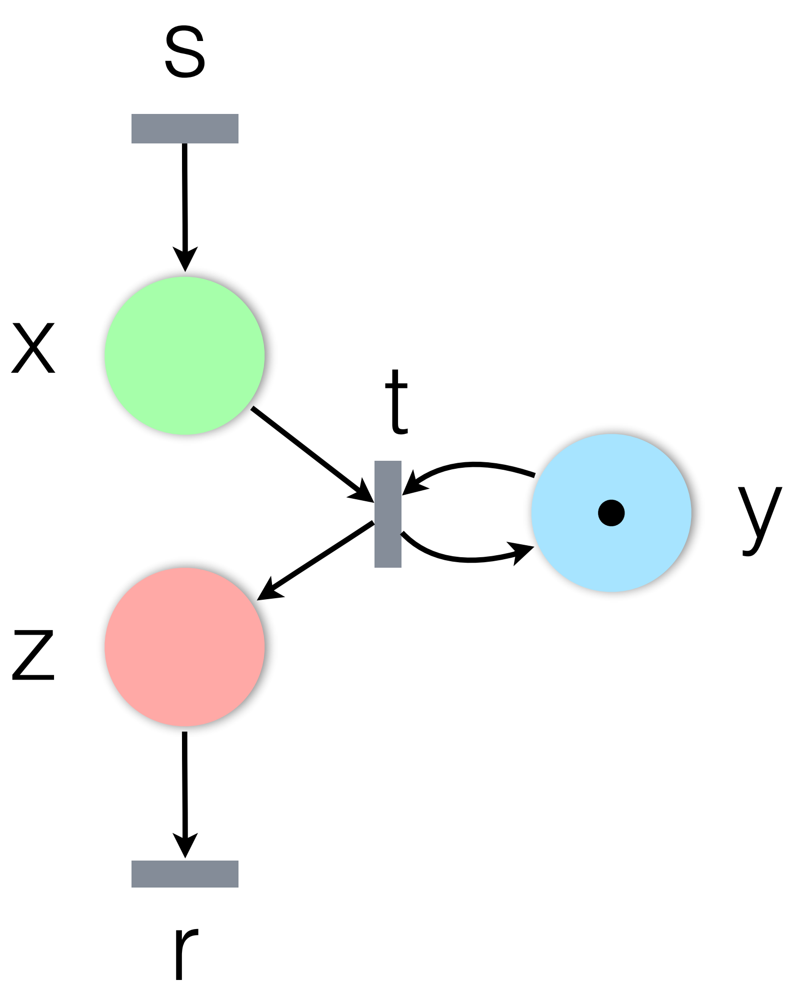


Ignore the order of transitions:

- **marking equation** [Murata '77]

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} s \\ t \\ r \end{bmatrix}$$

Reachable markings satisfy marking equation



Ignore the order of transitions:

- **marking equation** [Murata '77]

$$M = m_0 + C\,X$$

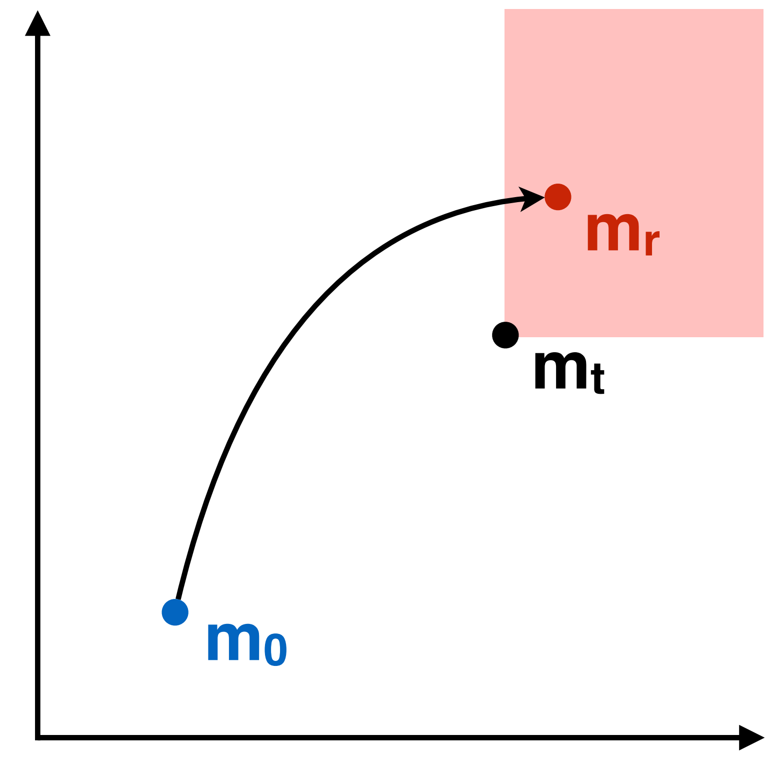
marking vector · initial marking · incidence matrix · transition vector

Coverability problem

Given a Petri net with:

- initial marking $\mathbf{m_0}$
- target marking $\mathbf{m_t}$

Is there a reachable marking $\mathbf{m_r}$ that **covers** $\mathbf{m_t}$?



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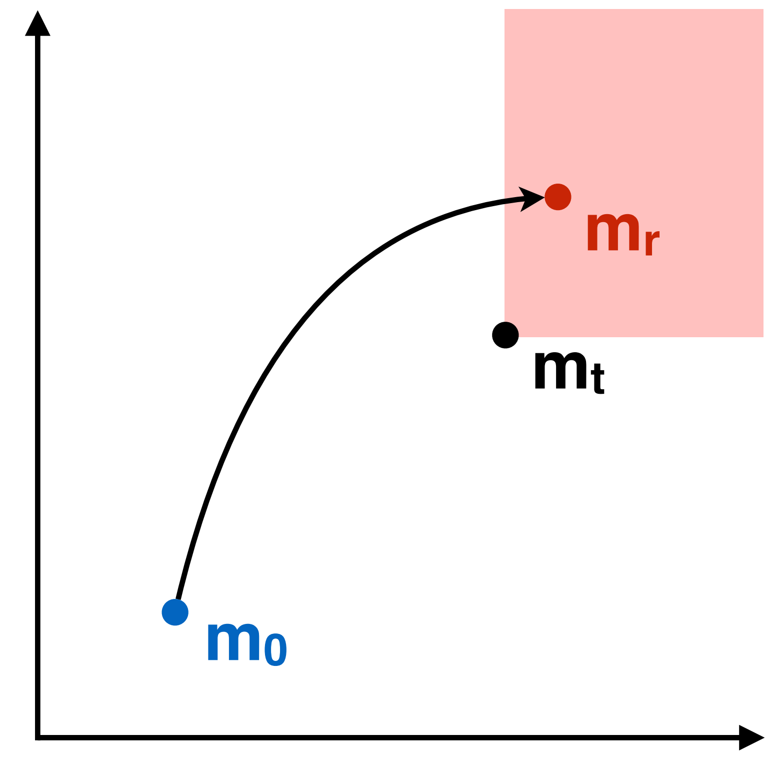
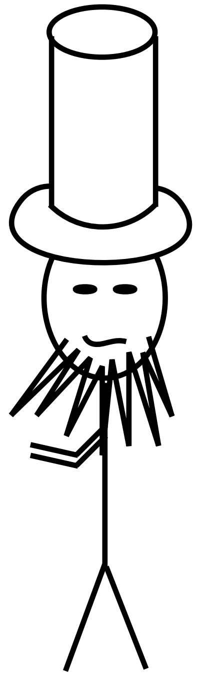


If $\mathbf{m_t}$ is **not coverable**, Petri net is **safe**.

Adding coverability constraint to marking equation yields basic LinCon

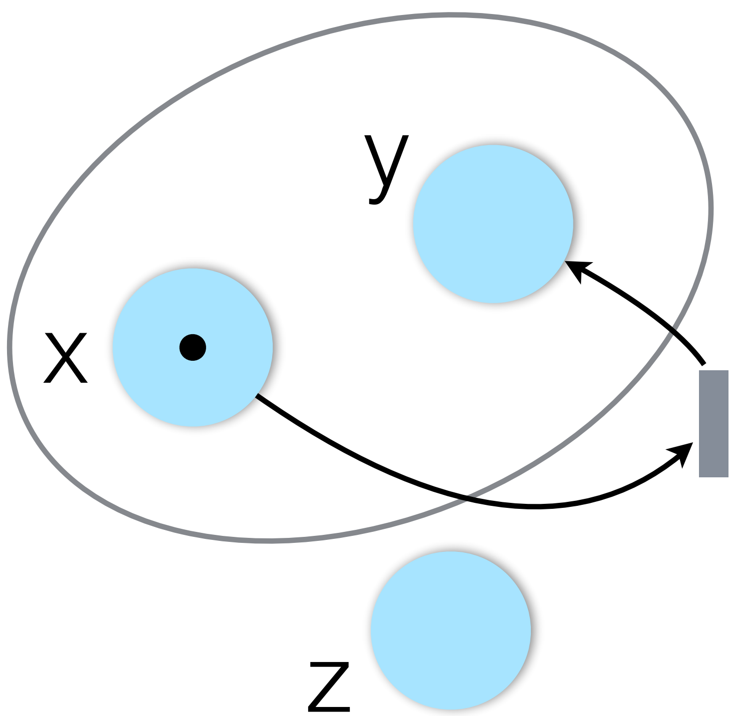
$$M = m_0 + CX$$
$$M \geq m_t$$
$$X \geq 0$$



If the **constraints are not feasible**, the **Petri net is safe**.

Strengthening LinCon using traps [EM '00]

Trap — **set of places** such that every **transition that consumes tokens** from it also **puts tokens** into it.



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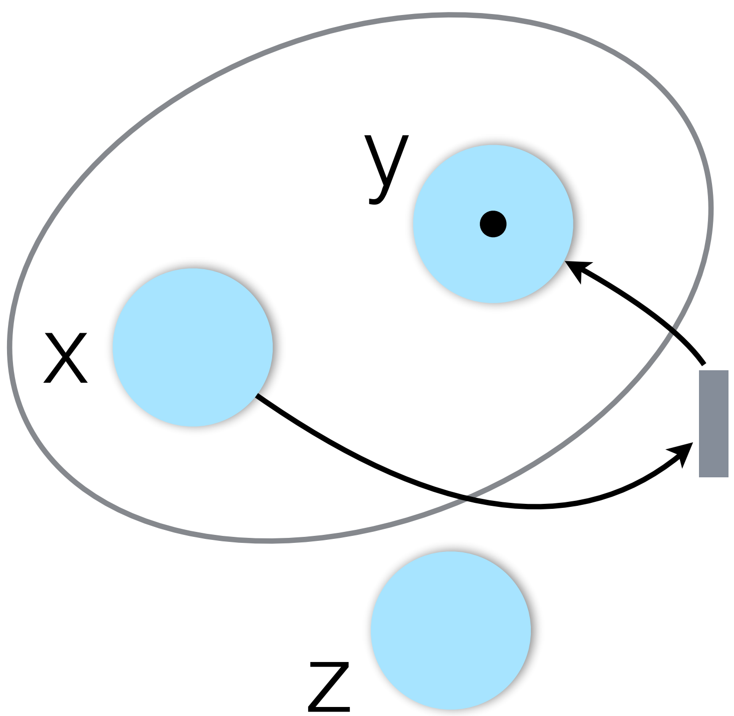

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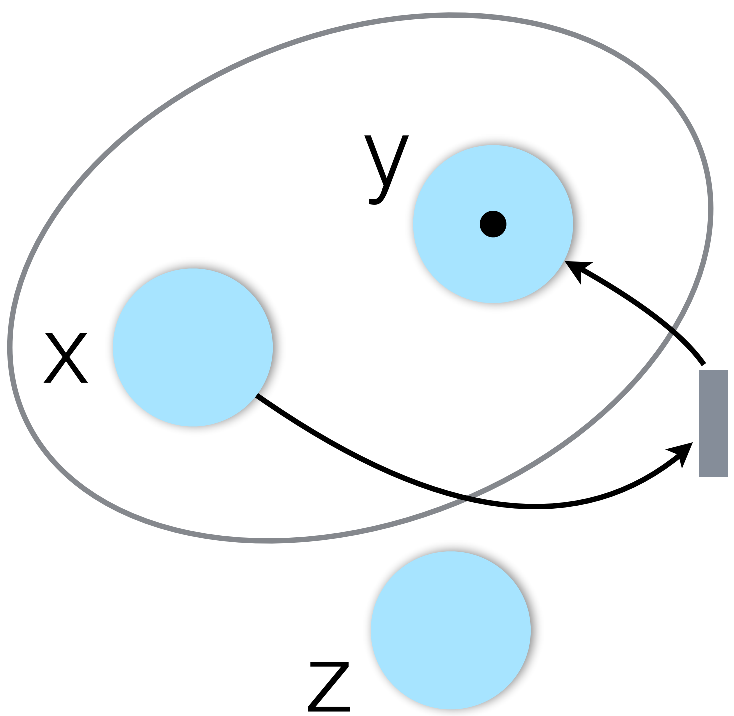

If a trap is marked, it stays marked.

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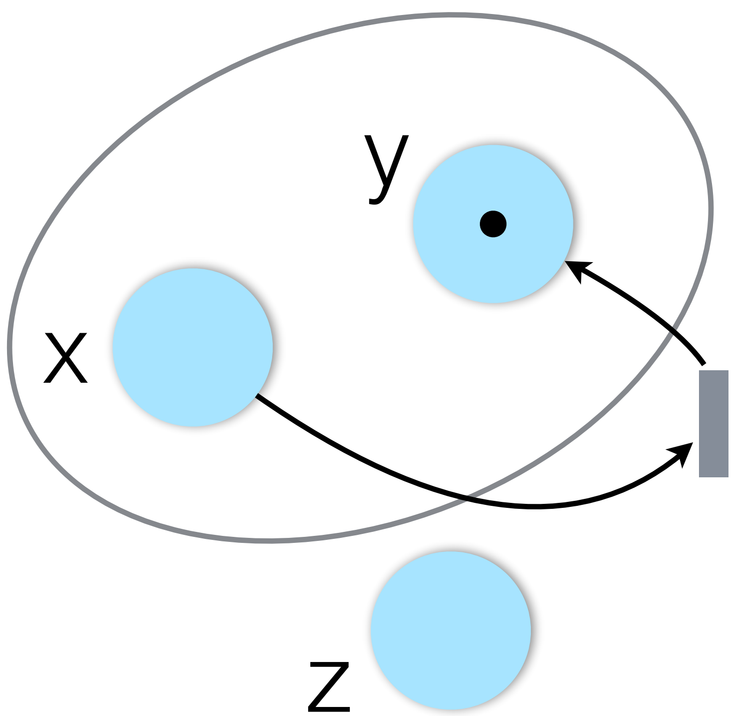

If a trap is marked, it stays marked.

$$x + y \geq 1$$

LinCon with traps [EM '00]

$$
\begin{aligned}
M &= m_0 + CX \\
M &\geq m_t \\
X &\geq 0
\end{aligned}
$$

LinCon with traps [EM '00]

$$M = m_0 + CX$$
$$M \geq m_t$$
$$X \geq 0$$

$\longrightarrow$ no solution

safe

LinCon with traps [EM '00]

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no solution
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$$M = m_r$$
$$X = x_r$$

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no solution

safe

$$M = m_r$$
$$X = x_r$$

Is there a trap

- initially marked
- empty at m_r

LinCon with traps [EM '00]

$$M = m_0 + CX$$
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no solution

safe

$$M = m_r$$
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SAT query:

Is there a trap

- initially marked
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no solution
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SAT query:
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no solution
inconclusive

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no solution
inconclusive

$$T_p = \begin{cases} 1, & p \text{ in trap} \\ 0, & \text{otherwise} \end{cases}$$

LinCon with traps [EM ’00]

$$\begin{aligned} M &= m_0 + CX \\ M &\geq m_t \\ X &\geq 0 \\ T^{\tau} M &\geq 1 \end{aligned}$$

no solution
safe

$$M = m_r$$
$$X = x_r$$

SAT query:

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no solution
inconclusive

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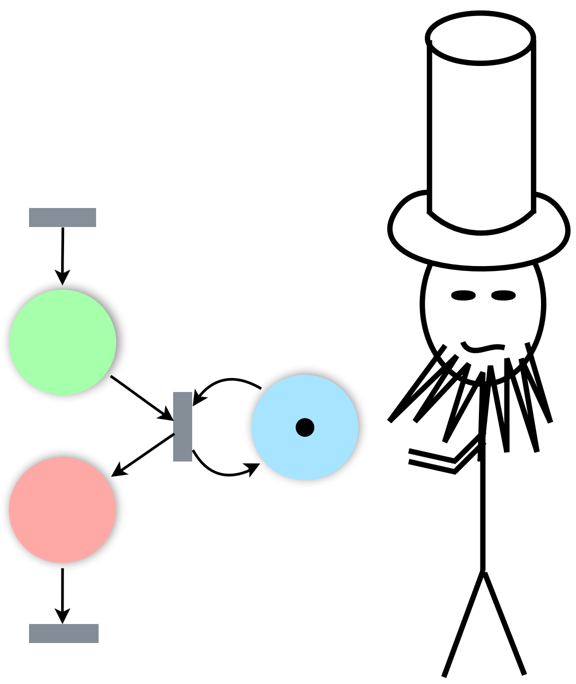

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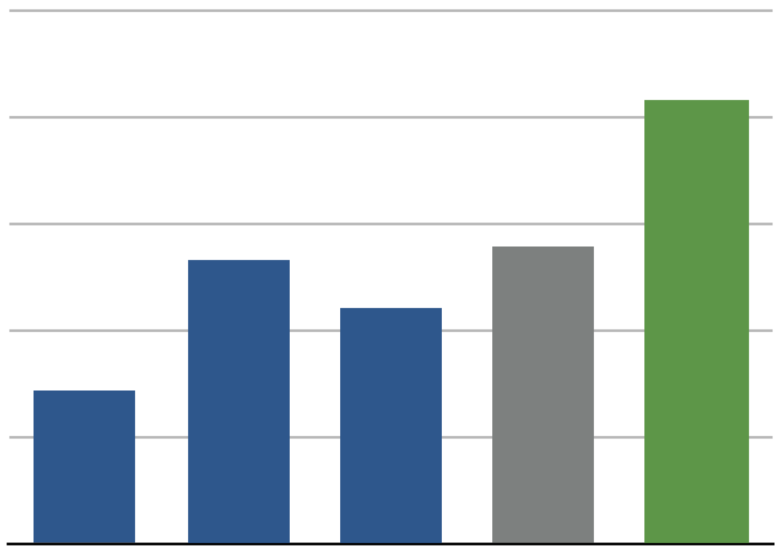

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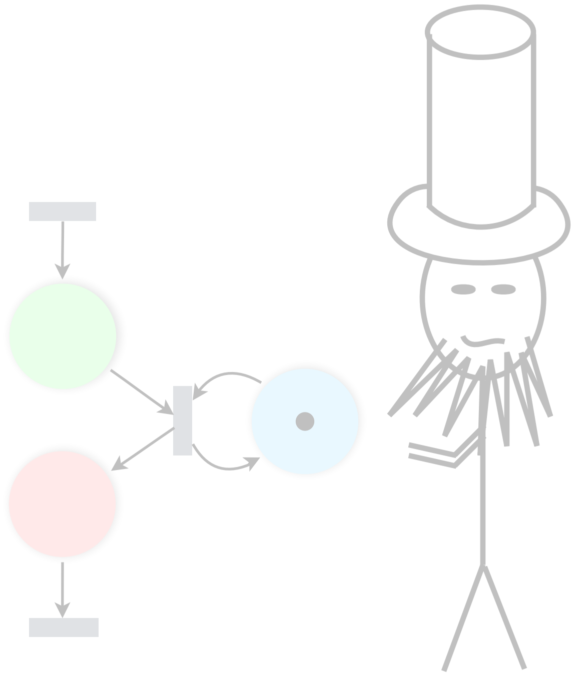

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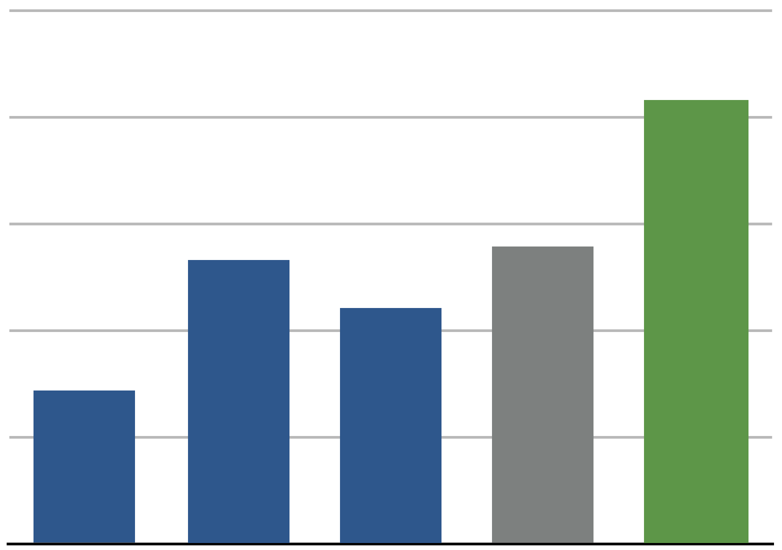


Experiments

The origin of examples

- MIST — https://github.com/pierreganty/mist

  Examples from the literature

- BFC — http://www.cprover.org/bfc/

  Examples from verification of concurrent C programs

- Provenance verification for message-passing programs [MMW '13]

  Examples modeling a medical system and a bug-tracking system

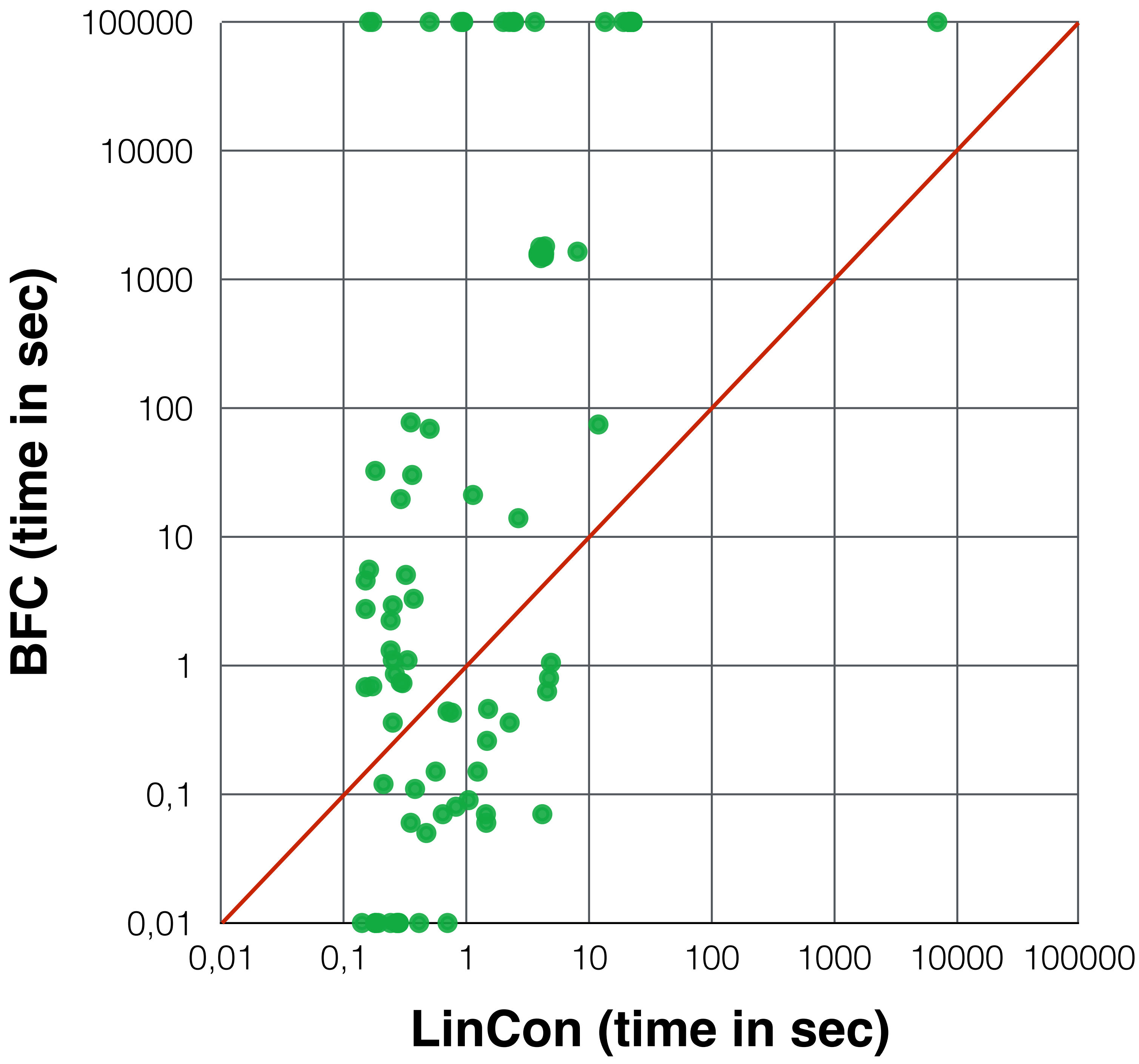
- SOTER — http://mjolnir.cs.ox.ac.uk/soter/ [DKO '13]

  Examples from verification of Erlang programs
  Contains a Petri net with **66,950 places** and **213,625 transitions**

Main point here:

LinCon works well even without traps

LinCon without traps is fast



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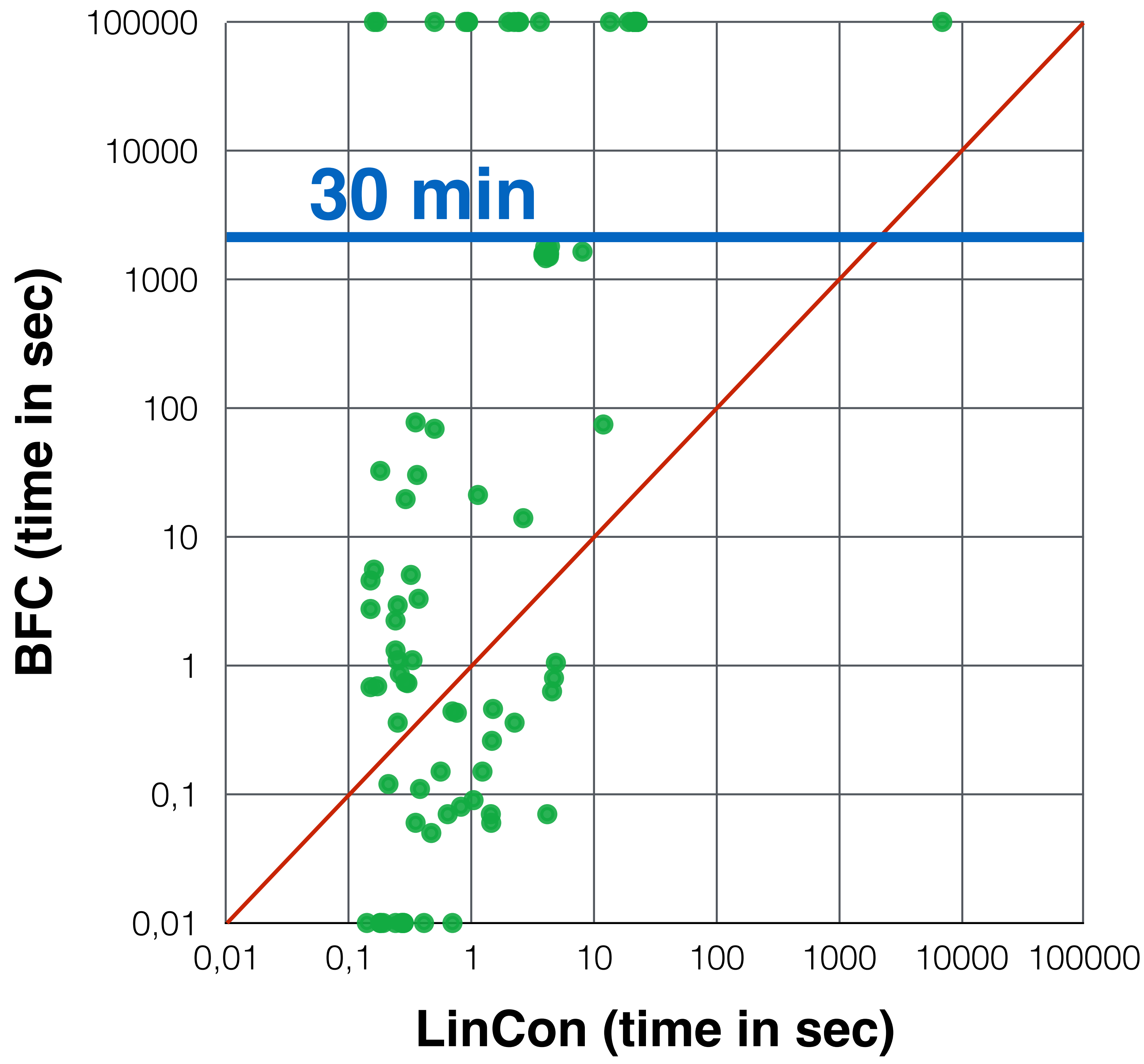

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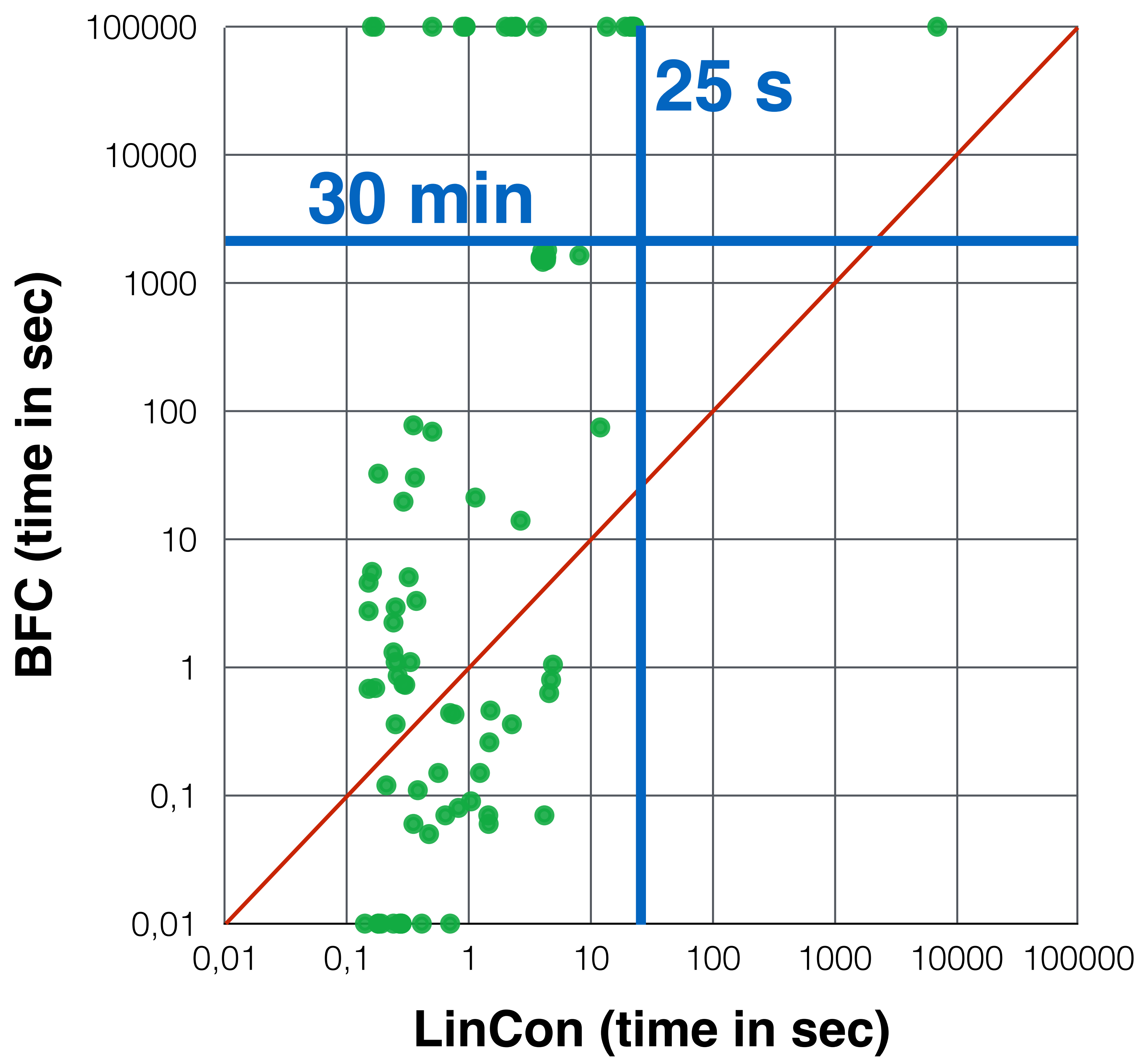

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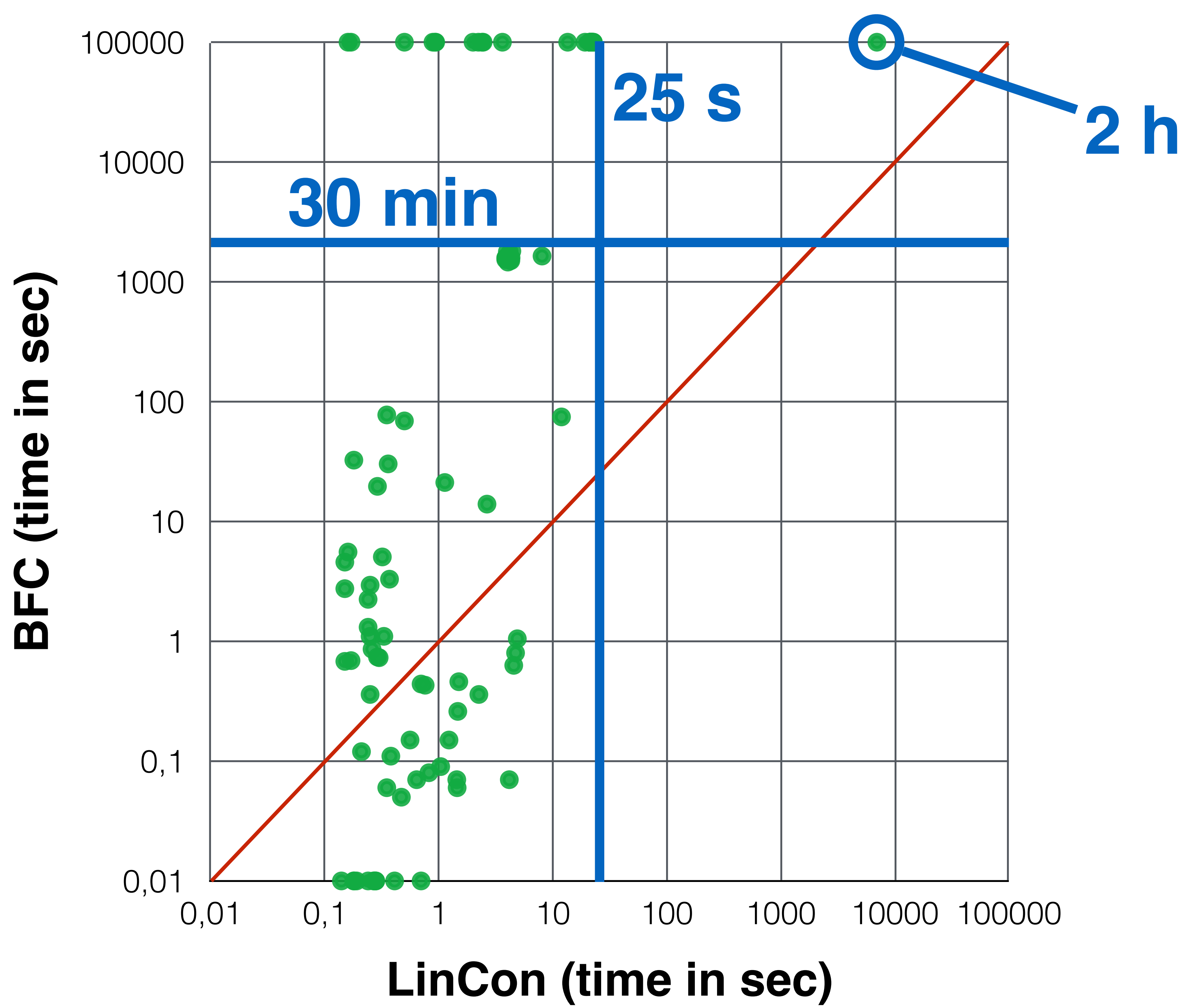
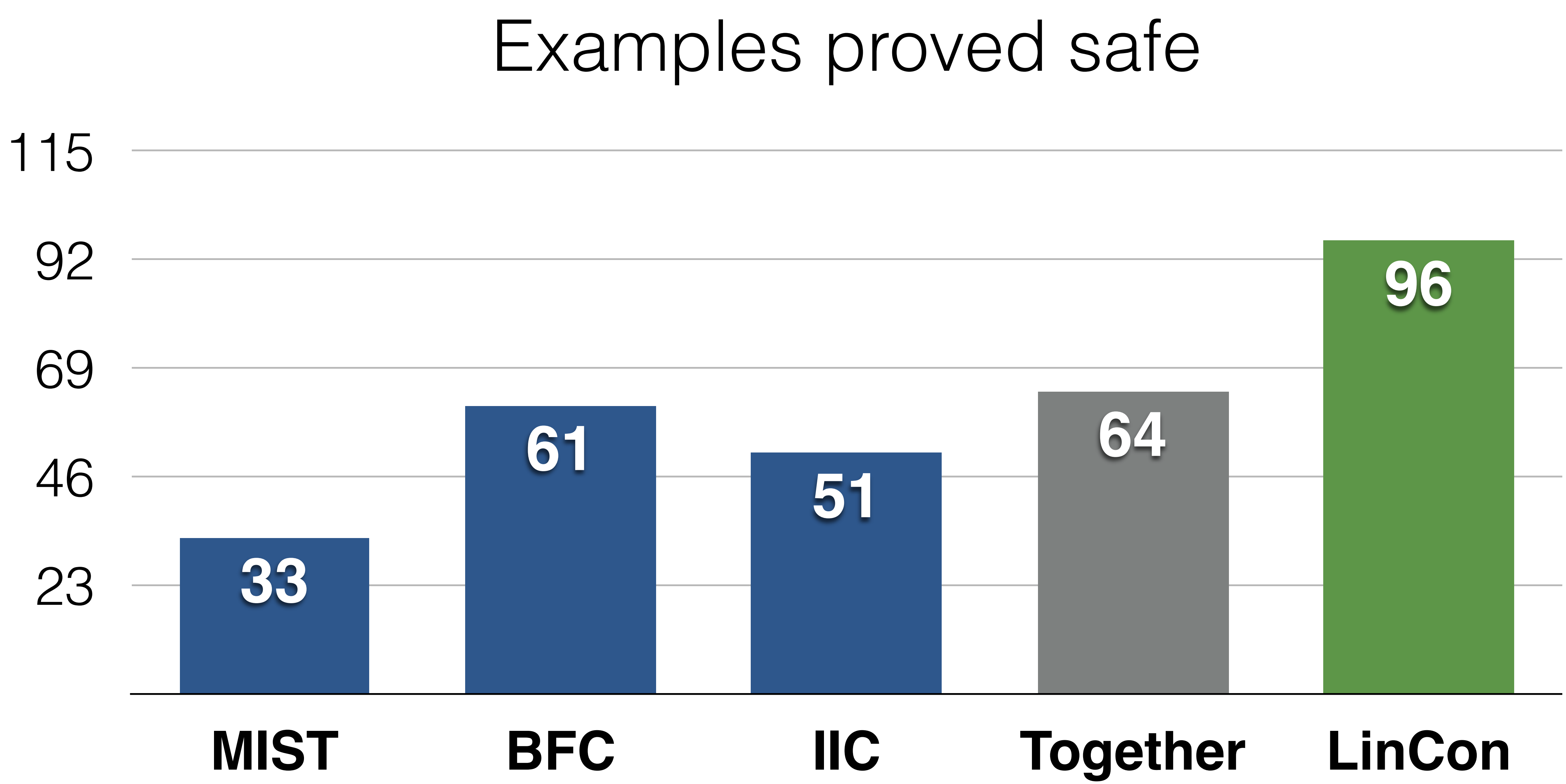


LinCon is “quite complete”



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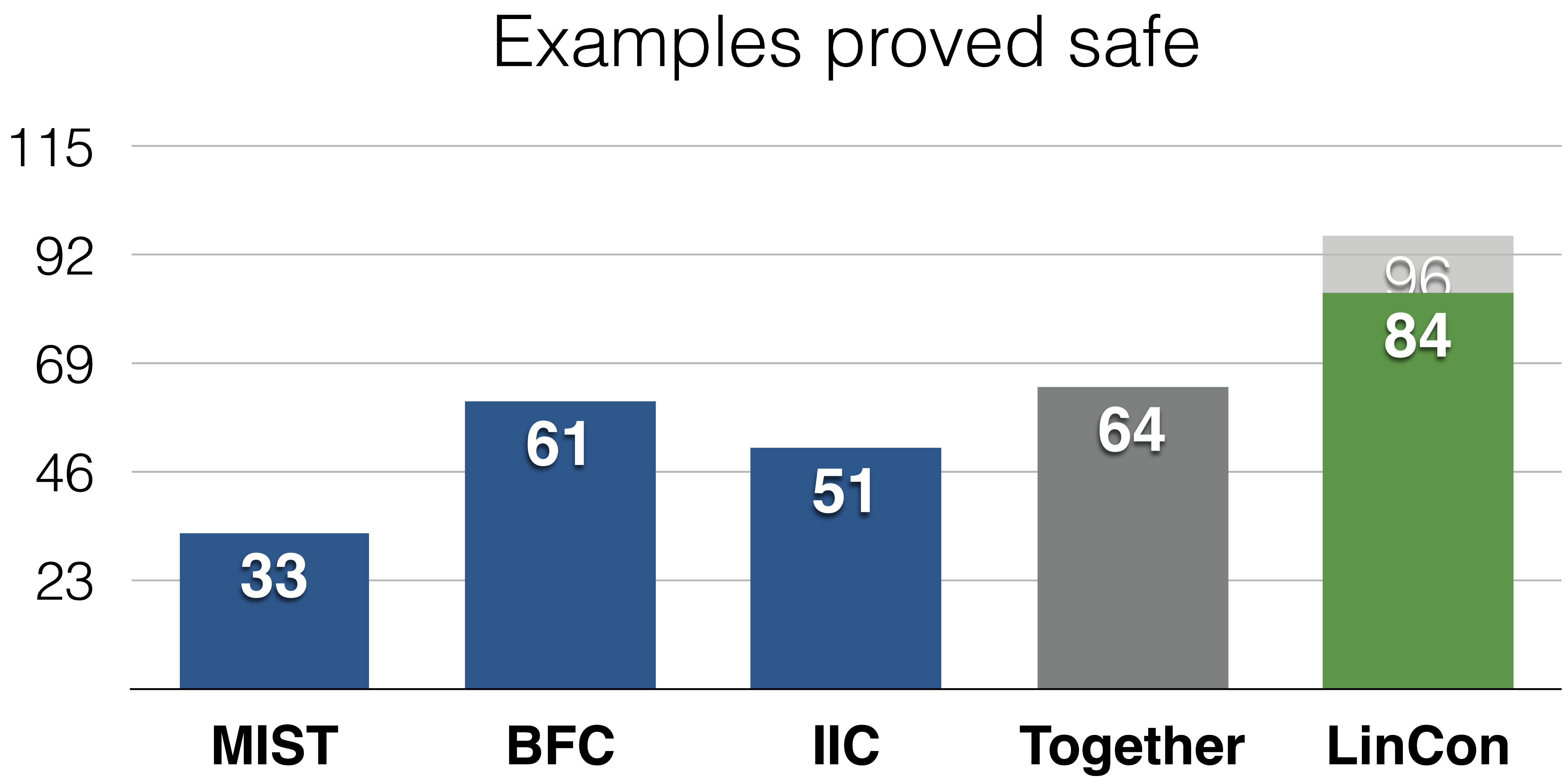
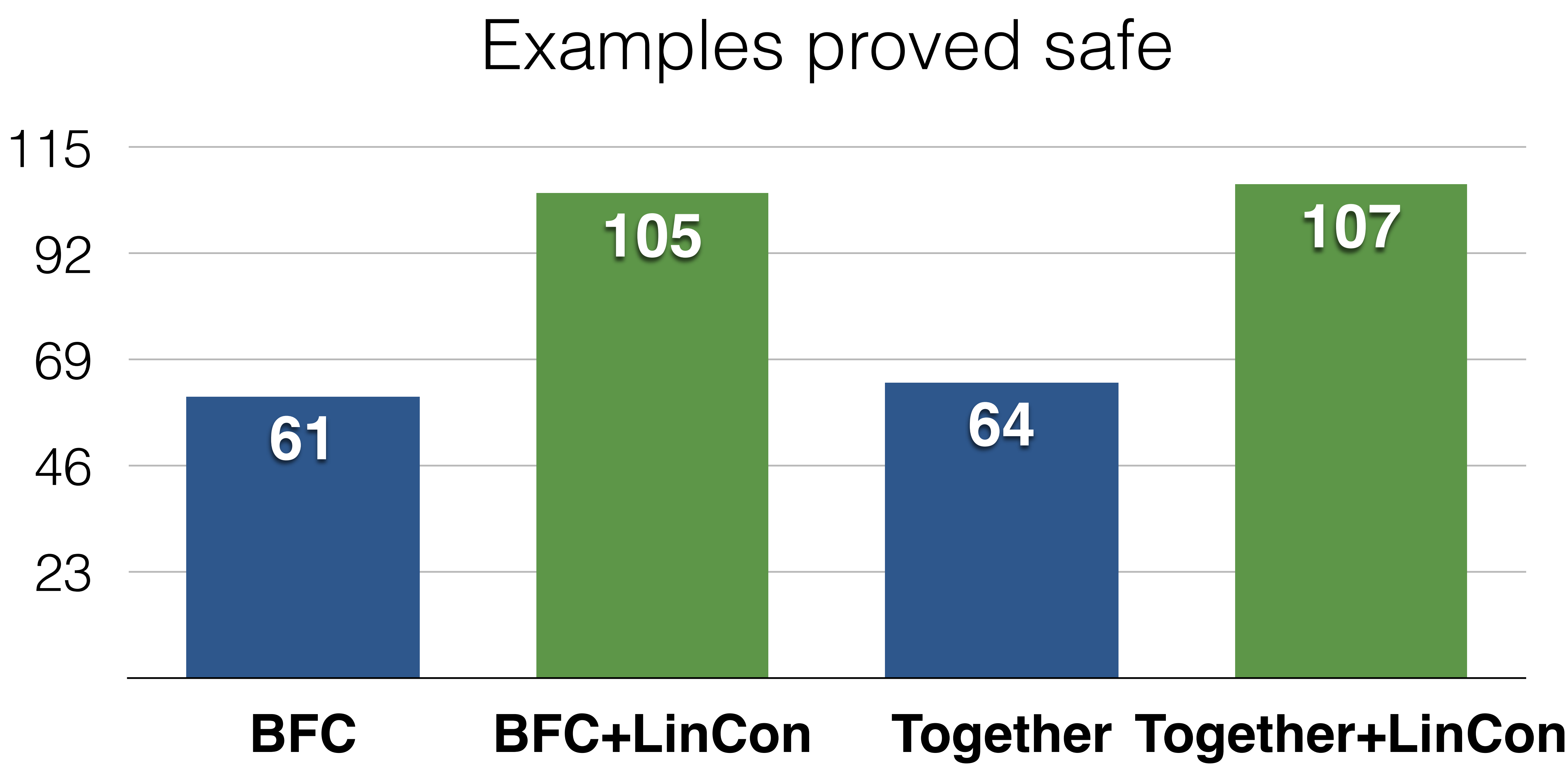


If LinCon were combined with other tools



Summary

- We've revisited a **linear constraint approach** to **Petri net coverability**
- LinCon is **incomplete**, but **useful**

  … on its own

  … as a cheap preprocessing step in other tools