Technische Universität München **Summer term 2017**
I7
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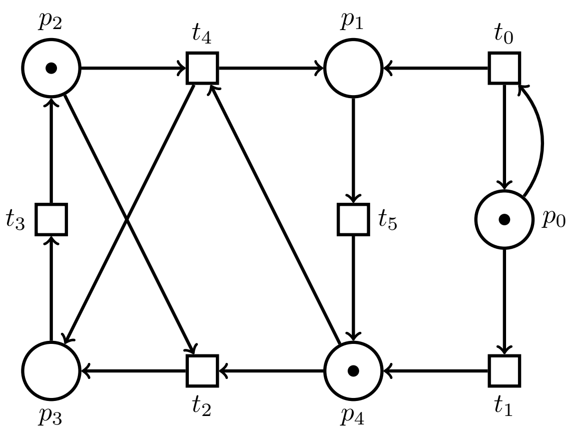
Petri nets — Homework 7

Due 26.07.2017

Exercise 7.1
Consider the following free-choice system $(\mathcal{N}, M_0)$:



(a) Give all minimal proper siphons of $(\mathcal{N}, M_0)$.

(b) Use (a) to say whether $(\mathcal{N}, M_0)$ is live or not.

Exercise 7.2

(a) Exhibit a non live system $(\mathcal{N}, M_0)$ for which every proper siphon contains a trap marked at M_0.

(b) Exhibit a live system $(\mathcal{N}, M_0)$ with a proper siphon that does not contain a trap marked at M_0. [Hint:]

(c) A system $(\mathcal{N}, M_0)$ is monotonously live if $(\mathcal{N}, M)$ is live for every marking $M \geq M_0$. In exercise #2.1, we have seen that Petri nets are generally not monotonously live. Show that live free-choice systems are monotonously live.

Exercise 7.3
Let $(\mathcal{N}, M_0)$ be a bounded and strongly connected free-choice system which is deadlock-free, where $\mathcal{N} = (P, T, F)$. For every $M \in \mathbb{N}^P$, let $d(M)$ be the number of transitions dead at M. Let $K \in \mathbb{N}^P$ be such that $d(K) = \max\{d(M) : M_0 \xrightarrow{*} M\}$.

(a) Let $u \in T$ be a transition not dead at K. Show that there exists an infinite firing sequence $\sigma \in T^\omega$ enabled at K and containing infinitely many occurrences of u. [Hint:]

(b) Let $u, v \in T$ be such that u is not dead at K and $v \in (u^\bullet)^\bullet$. Show that v is not dead at K. [Hint:]

(c) Show that there exists a path $\gamma \in (T \cup P)^*$ of $\mathcal{N}$ such that γ contains all transitions of T and γ starts with a transition enabled at K.

(d) Use (b) and (c) to show that $d(K) = 0$, and hence that $(\mathcal{N}, M_0)$ is live.

Exercise 7.4

Exercise #7.1 shows the following theorem:

> Every bounded, strongly connected and free-choice system is live if and only if it is deadlock-free.

Show that this theorem does not hold anymore if we remove any of its three conditions. More precisely,

(a) Exhibit a bounded and free-choice system which is deadlock-free, but not live;

(b) Exhibit a bounded and strongly connected system which is deadlock-free, but not live;

(c) ★ Exhibit a strongly connected and free-choice system which is deadlock-free, but not live. [Hint:]