Exercise 6.1

(a) Give an $S$-system $(\mathcal{N}, M_0)$ that is 1-bounded and such that $|M_0| > 1$.
(b) Give a strongly connected $T$-system $(\mathcal{N}, M_0)$ which is not live and such that $M_0 \neq 0$.
(c) Give a bounded $T$-system $(\mathcal{N}, M_0)$ which is not strongly connected and such that $M_0 \neq 0$.
(d) Let $(\mathcal{N}, M_0)$ be a $T$-system. Show that if $(\mathcal{N}, M_0)$ is strongly connected and live, then it is bounded.
(e) ★ Reprove (d), but this time without assuming that $(\mathcal{N}, M_0)$ is live.

Exercise 6.2

(a) Let $\mathcal{N} = (P,T,F)$ be a Petri net, and let $s,t \in T$ be such that $\bullet s \cap \bullet t = \emptyset$. Show that if $M \xrightarrow{ts} M'$, then $M \xrightarrow{st} M'$.
(b) Let $\mathcal{N} = (P,T,F)$ be a Petri net which is not strongly connected. Show that $P \cup T$ can be partitioned into two disjoint sets $U, V \subseteq P \cup T$ such that $F \cap (V \times U) = \emptyset$.
(c) Let $U$ and $V$ be a partition as in (b). Show that if $M \xrightarrow{\sigma} M'$, then there exist $\sigma_U \in (T \cap U)^*$ and $\sigma_V \in (T \cap V)^*$ such that $\sigma = \sigma_U \sigma_V$ and $M \xrightarrow{\sigma_U \sigma_V} M'$.
(d) Let $(\mathcal{N}, M_0)$ be live and bounded. Use (a), (b) and (c) to show that $\mathcal{N}$ is strongly connected.

Exercise 6.3

(a) Show that the problem of determining whether a $T$-system is not live belongs to NP.
(b) Give a polynomial time algorithm for deciding liveness of $T$-systems.
(c) Test whether the following $T$-system is live by using your previous algorithm: