Technische Universität München **Summer term 2017**
I7
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Petri nets — Homework 6

Due 12.07.2017

Exercise 6.1

(a) Give an S-system $(\mathcal{N}, M_0)$ that is 1-bounded and such that $|M_0| > 1$.

(b) Give a strongly connected T-system $(\mathcal{N}, M_0)$ which is not live and such that $M_0 \neq \mathbf{0}$.

(c) Give a bounded T-system $(\mathcal{N}, M_0)$ which is not strongly connected and such that $M_0 \neq \mathbf{0}$.

(d) Let $(\mathcal{N}, M_0)$ be a T-system. Show that if $(\mathcal{N}, M_0)$ is strongly connected and live, then it is bounded.

(e) ★ Reprove (d), but this time without assuming that $(\mathcal{N}, M_0)$ is live.

Exercise 6.2

(a) Let $\mathcal{N} = (P, T, F)$ be a Petri net, and let $s, t \in T$ be such that ${}^\bullet s \cap t^\bullet = \emptyset$. Show that if $M \xrightarrow{ts} M'$, then $M \xrightarrow{st} M'$.

(b) Let $\mathcal{N} = (P, T, F)$ be a Petri net which is not strongly connected. Show that $P \cup T$ can be partitioned into two disjoint sets $U, V \subseteq P \cup T$ such that $F \cap (V \times U) = \emptyset$.

(c) Let U and V be a partition as in (b). Show that if $M \xrightarrow{\sigma} M'$, then there exist $\sigma_U \in (T \cap U)^*$ and $\sigma_V \in (T \cap V)^*$ such that $\sigma = \sigma_U \sigma_V$ and $M \xrightarrow{\sigma_U \sigma_V} M'$.

(d) Let $(\mathcal{N}, M_0)$ be live and bounded. Use (a), (b) and (c) to show that $\mathcal{N}$ is strongly connected.

Exercise 6.3

(a) Show that the problem of determining whether a T-system is *not live* belongs to NP.

(b) Give a polynomial time algorithm for deciding liveness of T-systems.

(c) Test whether the following T-system is live by using your previous algorithm: