Exercise 5.1
Consider the following Petri net $\mathcal{N} = (P, T, F)$:

(a) Give a basis of the vector space of $T$-invariants of $\mathcal{N}$. [Hint: use a characterization of $T$-invariants.]

(b) Let $M = \{p_1, p_2, p_4, p_4\}$ and $M' = \{p_1, p_3, p_5\}$. We have shown that $\mathcal{N}$ is bounded from any initial marking in #4.3(b). Can you tell whether $(\mathcal{N}, M)$ and $(\mathcal{N}, M')$ are live?

Exercise 5.2

(a) Show that the union of two traps is also a trap.

(b) Give an algorithm that takes as input a Petri net $\mathcal{N} = (P, T, F)$ and a subset $R \subseteq P$, and that returns the largest trap of $\mathcal{N}$ contained in $R$.

(c) Show that your algorithm is correct and that it always terminates.
Exercise 5.3
Consider the following Petri net \( N \):

(a) Use siphons/traps to prove or disprove that \( N \) is live from \( M_0 = \{ p_2, 3 \cdot p_4 \} \).

(b) Can the marking equation be used to prove or disprove that \( \{ p_2, p_4 \} \xrightarrow{\Delta} \{ p_1, 3 \cdot p_2, p_3 \} \)? Is so, why? If not, can traps or siphons help?

Exercise 5.4
Let \( N = (P, T, F) \) be a Petri net. Let \( Q_x = \{ p : p \in P, x_p = \text{true} \} \) for every \( x \in \{ \text{false}, \text{true} \}^P \).

(a) Give a Boolean formula \( \varphi \) over variables \( \{ x_p : p \in P \} \) such that \( \varphi(x) \) holds if and only if \( Q_x \) is a siphon of \( N \).

(b) Let \( M_0, M \in \mathbb{N}^P \). Modify you previous construction in order to obtain a formula \( \psi \) such that \( \psi(x) \) holds if and only if \( Q_x \) is a siphon of \( N \), \( M_0(Q_x) = 0 \) and \( M(Q_x) > 0 \).

(c) Construct \( \psi \) for the Petri net of #5.3 with \( M_0 = \{ p_1, p_4 \} \) and \( M = \{ p_1, 3 \cdot p_2, p_3 \} \).