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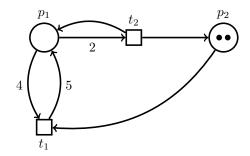
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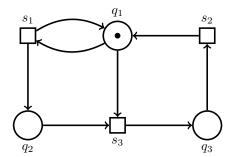
## Petri nets — Homework 3

Due 31.05.2017

## Exercise 3.1

Let  $\mathcal{N}$  and  $\mathcal{N}'$  be respectively the left and right Petri nets below.





Use the backward algorithm to answer the following questions.

- (a) Describe the set of initial markings from which  $\{p_2, p_2\}$  is coverable in  $\mathcal{N}$ . Illustrate this set.
- (b) Determine whether  $\{q_1, q_3\}$  is coverable from  $\{q_1\}$  in  $\mathcal{N}'$ .
- (c) Determine whether  $\{q_1, q_2\}$  is coverable from  $\{q_1\}$  in  $\mathcal{N}'$ .

## Exercise 3.2

A Petri net with reset, doubling and transfer arcs is a tuple (P, T, F, R, D, Tr) where (P, T, F) is a Petri net,

$$R \subseteq P \times T$$
,  $D \subseteq T \times P$ ,  $Tr \subseteq (P \times T) \cup (T \times P)$ ,

and F, R, D and Tr are pairwise disjoint. Let  $M \in \mathbb{N}^P$  and  $t \in T$ . We say that t is enabled at M if M(p) > 0 for every  $(p,t) \in F$ . Firing t at M has the following effect:

- every arc  $(p,t) \in F$  consumes a token from p;
- every arc  $(t, p) \in F$  produces a token in p;
- every arc  $(p,t) \in R$  empties p;
- every arc  $(t, p) \in D$  doubles the amount of tokens in p;
- every arc  $(p,t) \in Tr$  empties p;
- every arc  $(t,p) \in Tr$  adds  $\sum_{(q,t) \in Tr} M(q)$  tokens to p.

Show that the backward algorithm works for this extended class of Petri nets by showing that it is monotonic, i.e. show that for every markings  $X, X', Y' \in \mathbb{N}^P$ , if  $X \to Y$  and  $X' \ge X$ , then  $X' \to Y'$  for some  $Y' \ge Y$ .

## Exercise 3.3

- (a) Show that Petri nets with inhibitor arcs are not monotonic.
- (b) Give a Petri net with reset arcs  $\mathcal N$  and a marking M such that  $(\mathcal N,M)$  is bounded, but such that there exists a sequence  $M \xrightarrow{\sigma} M' \xrightarrow{\sigma'} M''$  with  $M'' \geq M'$  and  $M'' \neq M'$ .