Petri nets — Homework 2

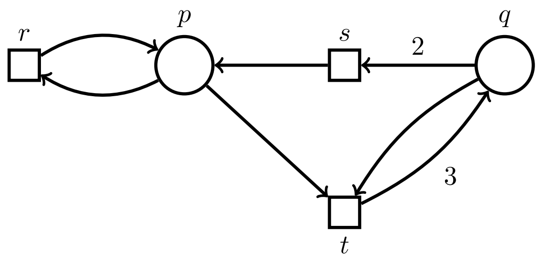
Due 17.05.2017

Exercise 2.1

(a) Give a Petri net $\mathcal{N}$ and two markings M and M' such that $M \leq M'$, $(\mathcal{N}, M)$ is bounded, and $(\mathcal{N}, M')$ is *not* bounded.

(b) Give a Petri net $\mathcal{N}$ and two markings M and M' such that $M \leq M'$, $(\mathcal{N}, M)$ is deadlock-free, and $(\mathcal{N}, M')$ is *not* deadlock-free.

(c) Consider the following Petri net $\mathcal{N}$ (with weighted arcs):



Give markings M and M' such that $M \leq M'$, $(\mathcal{N}, M)$ is live, and $(\mathcal{N}, M')$ is deadlock-free but *not* live.

Exercise 2.2

Let $\mathcal{N} = (P, T, W)$ be a Petri net with weighted arcs. Let $M, M' \in \mathbb{N}^P$, $\sigma, \sigma' \in T^*$ and $t \in T$ be such that $M \xrightarrow{\sigma t \sigma'} M'$. Prove or disprove the following statements:

(a) if t does not consume any token, i.e $W(p,t) = 0$ for every $p \in P$, then $M \xrightarrow{t \sigma \sigma'} M'$.

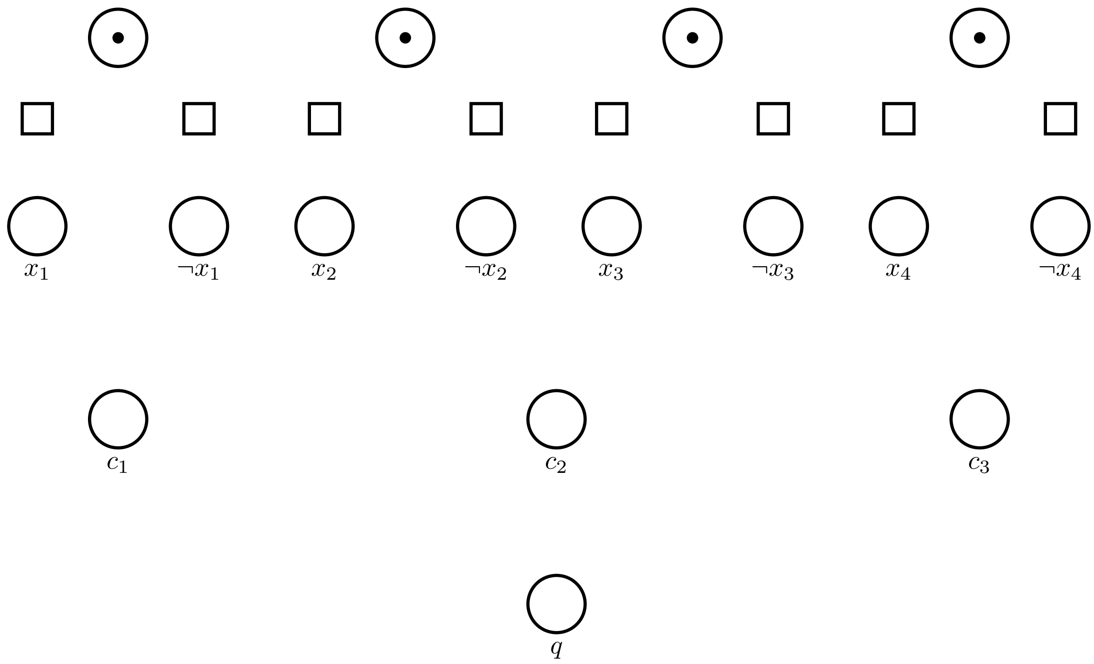
(b) if t consumes no more tokens than it produces, i.e $W(p,t) \leq W(t,p)$ for every $p \in P$, then $M \xrightarrow{t \sigma \sigma'} M'$.

(c) if t does not produce any token, i.e. $W(t,p) = 0$ for every $p \in P$, then $M \xrightarrow{\sigma \sigma' t} M'$.

(d) if t produces no more tokens than it consumes, i.e. $W(t,p) \leq W(p,t)$ for every $p \in P$, then $M \xrightarrow{\sigma \sigma' t} M'$.

Exercise 2.3

(a) Recall that 3-SAT is the problem of determining the satisfiabillity of a Boolean formula, in conjunctive normal form, whose clauses have at most three literals. It is well-known that 3-SAT is NP-complete. Give a polynomial time reduction from 3-SAT to Petri net coverability. You can simply illustrate your reduction for the formula $\varphi(x_1, x_2, x_3, x_4) = (x_1 \lor x_2 \lor x_4) \land (\neg x_1 \lor x_3 \lor \neg x_4) \land (\neg x_2 \lor \neg x_3 \lor \neg x_4)$ by extending the following partial Petri net in such a way that φ is satisfiable if and only if $\{q\}$ is coverable:



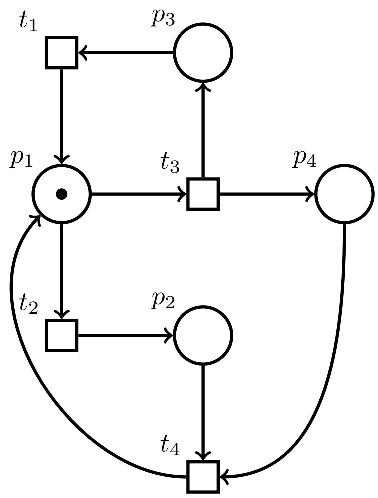
(b) Adapt your previous reduction to boundedness instead of coverability.

(c) ★ Give a polynomial time reduction from coverability to reachability. [Hint:]

(d) ★ Prove that the reduction you gave in (c) is correct. [Hint:]

Exercise 2.4

Consider the following Petri net $\mathcal{N} = (P, T, F)$:



(a) Draw a coverability graph for $(\mathcal{N}, \{p_1\})$.

(b) Is $(\mathcal{N}, \{p_1\})$ bounded? If so, why? If not, which places are bounded?

(c) Describe the set of markings coverable from $\{p_1\}$.

Exercise 2.5

The algorithm Coverability-Graph does not specify how the coverability graph should be traversed during its construction. Show that different traversal strategies can lead to different coverability graphs. More precisely, exhibit a marking M and two different coverability graphs for $(\mathcal{N}, M)$, where $\mathcal{N}$ is the following Petri net: