Definition 2.16 *Liveness and related properties*

$M' \in M \rightarrow$

A system is live if, for every reachable marking M and every transition t, there exists a marking $M' \in [M\rangle$ which enables t. If (N, M_0) is a live system, then we also say that M_0 is a live marking of N.

A system is place-live if, for every reachable marking M and every place s, there exists a marking $M' \in [M\rangle$ which marks s.

A system is deadlock-free if every reachable marking enables at least one transition; in other words, if no dead marking can be reached from the initial marking.

Definition 2.20 *Bounded systems, bound of a place*

A system is bounded if for every place s there is a natural number b such that $M(s) \leq b$ for every reachable marking M. If (N, M_0) is a bounded system, we also say that M_0 is a bounded marking of N.

The bound of a place s in a bounded system (N, M_0) is defined as

$$\max\{M(s) \mid M \in [M_0\rangle\}$$

A system is called b-bounded if no place has a bound greater than b.

bounded safe k-safe

↓

1-bounded

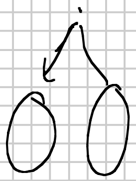
[illegible]

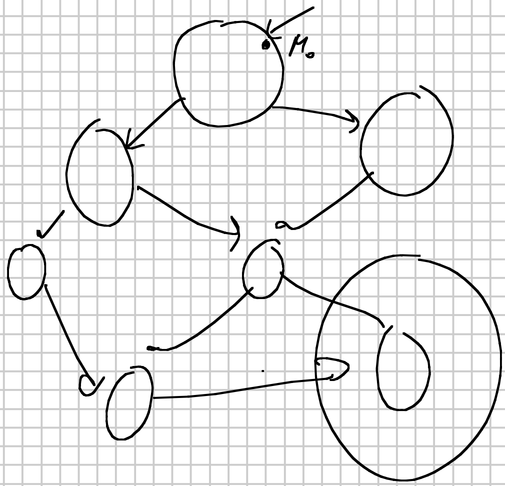
Definition 2.23 *Well-formed nets*

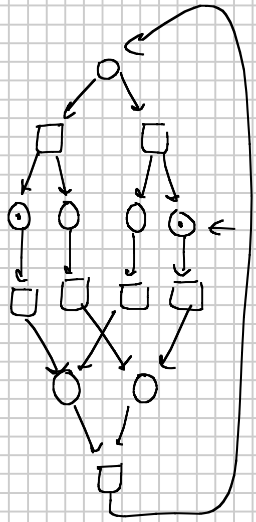
A net N is well-formed if there exists a marking M_0 of N such that (N, M_0) is a live and bounded system.

Lemma 2.24

Every live and bounded system (N, M_0) has a reachable marking M and an occurrence sequence $M \xrightarrow{\sigma} M$ such that all transitions of N occur in σ.







Theorem 2.25 *Strong Connectedness Theorem*

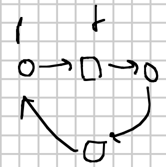
Well-formed nets are strongly connected.

Proof: If N is not strongly connected then there is an arc

$$x \to y$$

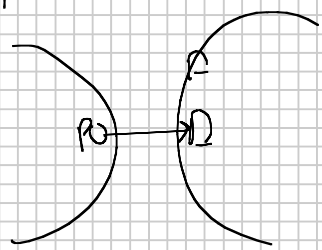
such that there is no path $y \to \cdots \to x$

Two cases:

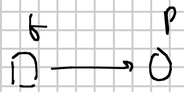


By liveness we can mark p again and again

Since there is no path from t to p we can do so without firing t



So p is not bounded.



By liveness we can fire t again and again

Since there is no path from p to t we can do so without firing any transition of $p^\bullet$

→ unboundedness.

Theorem 2.30 *A necessary condition for liveness*

If (N, M_0) is a live system, then every semi-positive S-invariant I of N satisfies $I \cdot M_0 > 0$.

Theorem 2.31 *A sufficient condition for boundedness*

Let (N, M_0) be a system. If N has a positive S-invariant I, then (N, M_0) is bounded.

Definition 2.32 *Markings that agree on all S-invariants*

Two markings M and L of a net are said to agree on all S-invariants if $I \cdot M = I \cdot L$ for every S-invariant I of the net.

Theorem 2.33 *A necessary condition for reachability*

Let (N, M_0) be a system, and let $M \in [M_0\rangle$. Then M and M_0 agree on all S-invariants.

Theorem 2.34 *Characterization of markings that agree on all S-invariants*

Two markings M and L of a net N agree on all S-invariants iff the equation $M + \mathbf{N} \cdot X = L$ has some rational-valued solution for X.

Proof:

($\Rightarrow$): Since M and L agree on all S-invariants, they also agree on a basis $\{I_1, \ldots, I_k\}$. For every vector I_j of this basis we have $I_j \cdot (L - M) = \mathbf{0}$. A well-known theorem of linear algebra states that the columns of $\mathbf{N}$ include a basis of the space of solutions of the homogeneous system

$$I_j \cdot X = \mathbf{0} \qquad (1 \leq j \leq k)$$

Therefore, $(L-M)$ is a linear combination in $\mathbb{Q}$ of these columns, i.e., $\mathbf{N} \cdot X = (L-M)$ has some rational-valued solution for X.

Theorem 2.38

Every well-formed net has a positive T-invariant.