Petri nets – Endterm

Last name: ____________________

First name: ____________________

Student ID no.: ____________________

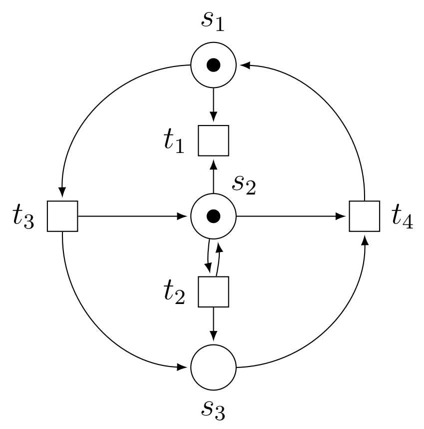
Signature: ____________________

- If you feel ill, let us know immediately.
- Please, **do not write** until told so.
- You will be given **90 minutes** to fill in all the required information and write down your solutions.
- Don't forget to **sign**.
- Write with a non-erasable **pen**, do not use red or green color.
- You are not allowed to use **auxiliary means** other than your pen.
- You may answer in **English or German**.
- Please turn off your **cell phone**.
- Should you require additional **scrap paper**, please tell us.
- You can obtain **40 points** in the exam. You need **17 points** in total to pass (grade 4.0).
- Don't fill in the table below.
- Good luck!

Ex 1	Ex 2	Ex 3	Ex 4	Ex 5	$\sum$

Exercise 1 **5P**

Construct the coverability graph of the Petri net below.



Exercise 2 **6P**

Reduce the coverability problem to the reachability problem.

For that, describe an algorithm that, given a Petri net (N, M_0) and a marking M, constructs a Petri net (N', M_0') and a marking M' such that M' is reachable in N' from M_0' **if and only if** M is coverable in N from M_0. The algorithm should run in polynomial time. You don't have to describe N' formally.

Give a brief argument showing that your construction is correct, i.e. show that if M is coverable in N from M_0, then M' is reachable in N' from M_0', and if M' is reachable in N' from M_0', then M is coverable in N from M_0.

Exercise 3 **9P=3+3+3**

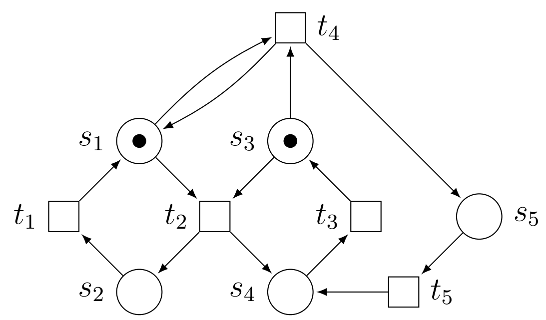
(a) Exhibit a net having a positive T-invariant but no positive S-invariant.

(b) Exhibit a net having a positive S-invariant but no positive T-invariant.

(c) Exhibit a net with a minimal siphon containing two input places of the same transition.

Exercise 4 **10P=3+3+4**

Consider the following Petri net:



(a) Give a basis of the space of S-invariants of the net.

(b) Find all three minimal traps of the net.

(c) Use (a) and (b) to show that s_2 and s_5 are mutually exclusive, i.e. there is no reachable marking M with $M(s_2) \geq 1$ and $M(s_5) \geq 1$.

Please turn over!

Exercise 5 **10P=3+3+4**

(a) Prove: If (N, M_0) is a live and bounded Petri net, then N has a positive T-invariant.

(b) Prove: If (N, M_0) is a live and bounded free-choice system and $M_0' \geq M_0$, then (N, M_0') is also live and bounded.

(c) Prove: Let N be a net and M a marking of N. The equation $M = \mathbf{N} \cdot X$ has a nonnegative integer solution $X : T \to \mathbb{N}$ **iff** there is marking M_0 of N such that $M + M_0$ is reachable from M_0 in N.

Note: $M + M_0$ is defined as the marking with $(M + M_0)(s) = M(s) + M_0(s)$.