Technische Universität München (I7)
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Petri nets – Homework 6

Discussed on Thursday 14th July, 2016.

For questions regarding the exercises, please send an email to meyerphi@in.tum.de or just drop by at room 03.11.042.

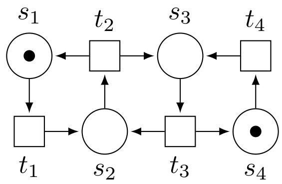
Exercise 6.1 Minimal traps and siphons in free-choice nets

A trap (resp. siphon) is *minimal* if it is proper (not empty) and contains no other proper trap (resp. siphon).

(a) Add arcs to the Petri net below such that it becomes a live and bounded free-choice system.

(b) Find all minimal traps and all minimal siphons of the resulting free-choice net.

(c) Does every minimal siphon contain a proper trap? Does every minimal trap contain a proper siphon?



Exercise 6.2 Characterization of minimal siphons

(a) Let N be a net, R a minimal siphon of N, and N_R the subnet generated by $(R, {}^\bullet R)$. Show: N_R is strongly connected.

Hint: For an arc (x, y) in N_R, with $Q = \{s \in R \mid \text{there exists a path from } s \text{ to } x \text{ in } N_R\}$, show that Q is a proper siphon, and therefore there exists a path from y into Q to X.

(b) Exhibit a strongly connected net in which not every place belongs to a minimal siphon.

Hint: Two places and two transitions suffice.

Exercise 6.3 Liveness and boundedness in free-choice systems

The result from the previous exercise can be used to show the following proposition (full proof given in Proposition 5.4 of "Free Choice Petri Nets" by J. Desel and J. Esparza):

Proposition 6.3.1. Let N be a well-formed free-choice net and R be a minimial siphon of N. Then

(1) R is a trap of N.

(2) The subnet generated by $(R, {}^\bullet R)$ is an S-component of N.

Using the above proposition, as well as Commoner's Liveness Theorem and Hack's Boundedness Theorem, prove or disprove the following:

(a) A bounded free-choice system (N, M_0) is live iff every minimal siphon of N is a trap marked at M_0.

(b) A live free-choice system (N, M_0) is bounded iff every minimal siphon of N is a trap marked at M_0.

Exercise 6.4 Reducing SAT to reachability in free-choice systems

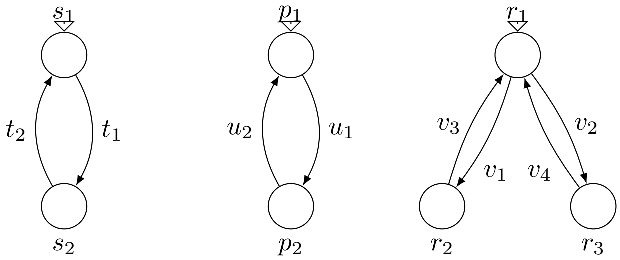
Reduce the satisfiability problem for boolean formulas in conjunctive normal form to the reachability problem in free-choice systems.

For that, give a polynomial time translation that, for a given formula φ, produces a free-choice system (N, M_0) and a marking M such that φ is satisfiable iff M is reachable in (N, M_0). Describe your reduction informally and give the resulting Petri net when applying it to the formula below.

$$\varphi = (\neg x_1 \vee \neg x_2 \vee x_3) \wedge (x_2 \vee \neg x_3)$$

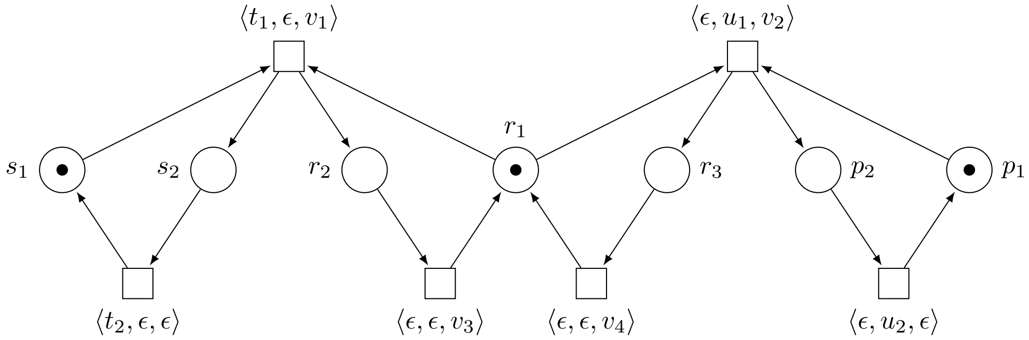
Exercise 6.5 **Unfoldings**

Consider the transition systems below, with the synchronization constraint $\mathbf{T}$:



$$\mathbf{T} = \{\langle t_1, \epsilon, v_1 \rangle, \langle t_2, \epsilon, \epsilon \rangle, \langle \epsilon, u_1, v_2 \rangle, \langle \epsilon, u_2, \epsilon \rangle, \langle \epsilon, \epsilon, v_3 \rangle, \langle \epsilon, \epsilon, v_4 \rangle\}$$

The following Petri net represents the product of the transition systems:



Using the search strategy $[w] \prec [w'] \Leftrightarrow |w| < |w'|$ for Mazurkiewicz traces w, w', compute the finite and complete prefix of the unfolding of this product.