Petri nets – Homework 5

Discussed on Wednesday 29th June, 2016.

For questions regarding the exercises, please send an email to meyerphi@in.tum.de or just drop by at room 03.11.042.

Exercise 5.1 Boundedness and liveness in S/T-systems

Show the following:

(a) An S-system (N, M_0) is bounded for any M_0.

(b) If (N, M_0) is a live S-system and $M_0' \geq M_0$, then (N, M_0') is also live.

(c) If (N, M_0) is a live and bounded T-system, then (N, M_0') is also bounded for any M_0'.

(d) If (N, M_0) is a live T-system and $M_0' \geq M_0$, then (N, M_0') is also live.

Exhibit Petri nets for the following:

(e) Give a bounded T-system (N, M_0) and a marking $M_0' \geq M_0$ such that (N, M_0') is not bounded.

(f) Give a 1-bounded S-system (N, M_0) where $M_0(S) > 1$.

(g) Give a live and 1-bounded T-system (N, M_0) with a circuit γ where $M_0(\gamma) > 1$.

Exercise 5.2 Marking equation in S-systems

In the lecture, it was shown that for an S-system (N, M_0), a marking M of N is reachable from M_0 iff the marking equation $M = M_0 + \mathbf{N} \cdot X$ has a nonnegative integer solution, i.e. $X : T \to \mathbb{N}$.

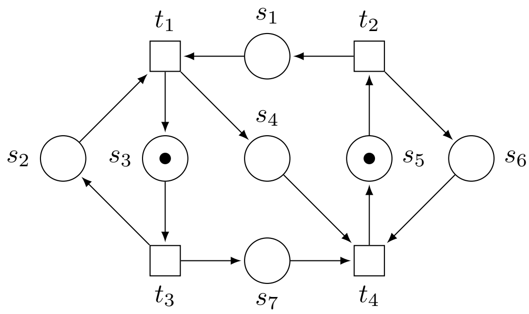
Show the following: For an S-system (N, M_0), a marking M of N is reachable from M_0 iff the marking equation $M = M_0 + \mathbf{N} \cdot X$ has a nonnegative rational solution, i.e. $X : T \to \mathbb{Q}$ with $X \geq 0$.

Note: We have not found a simple, constructive proof, so finding one is probably not that easy, though you should try to see the rationale for why this works. If you find an easy proof, please send it to us.

Exercise 5.3 Polynomial time algorithm for deciding liveness of a T-system

(a) Give a polynomial time algorithm to check if a T-system is live (note that a T-net may have an exponential number of circuits, so simply enumerating all circuits is infeasible).

(b) Apply your algorithm to the T-system below to decide if it is live.



Exercise 5.4 Strong Connectedness Theorem

Let (N, M_0) be a live and bounded Petri net. Show that N is strongly connected.

Hint: To show that the net is strongly connected, you need to show that for every arc $(x, y) \in F$, there is a path from y to x. Use liveness to construct a firing sequence containing the transition of the arc often enough and then use boundedness on the place of the arc to show that there needs to be a path back. You may also use the following lemma, proven in exercise 1.6:

Lemma 5.4.1 (Exchange Lemma)**.** Let u and v be transitions of a net satisfying ${}^\bullet u \cap v^\bullet = \emptyset$. If $M \xrightarrow{vu} M'$ then $M \xrightarrow{uv} M'$.