Petri nets – Homework 4

Discussed on Thursday 16th June, 2016.

For questions regarding the exercises, please send an email to meyerphi@in.tum.de or just drop by at room 03.11.042.

Exercise 4.1 S-invariants and T-invariants

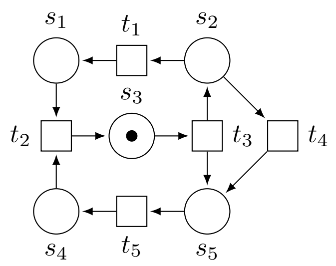
Four each of the following four invariants, check if the net below has such an invariant. If yes, give one such invariant. Can you make any statements about the liveness and boundedness of the net based on the existance of these invariants?

(a) a semi-positive S-invariant

(b) a semi-positive T-invariant

(c) a positive S-invariant

(d) a positive T-invariant



Exercise 4.2 Properties of invariants

Exhibit counterexamples that disprove the following conjectures:

(a) For a Petri net (N, M_0), an S-invariant I of N and a marking M, if $I \cdot M_0 = I \cdot M$, then M is reachable from M_0.

(b) For a Petri net (N, M_0) and a place s of N, if s is bounded, then there is a place invariant I of N with $I(s) > 0$.

(c) For a net N, if N has a positive transition invariant J, then it is well-formed (there is a marking M_0 such that (N, M_0) is live and bounded).

Exercise 4.3 Encoding traps into SAT

(a) Give a procedure that, given a net N, constructs a boolean formula φ satisfying the following properties:

- The formula contains variables r_s for each place $s \in S$,
- if φ is satisfiable, then N has a trap,
- and if φ is not satisfiable, then N has no trap.
- Additionally, if A is a model of φ, then the set given by $R = \{s \mid A(r_s)\}$ is a trap of N.

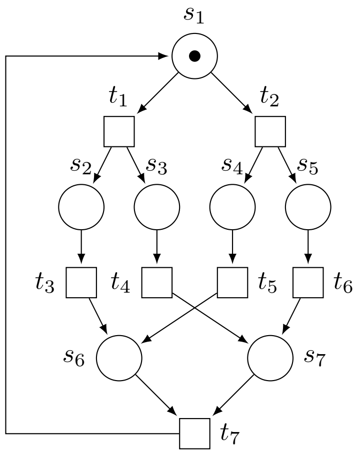
(b) Apply your procedure to the Petri net on the left below and give the resulting constraints.

(c) Adapt your procedure such that, given two marking M_0 and M, it adds additional constraints to ensure that any trap R obtained as a solution by the constraints is marked at M_0 and unmarked at M. The constraints should be satisfiable iff a trap marked at M_0 and unmarked at M exists.

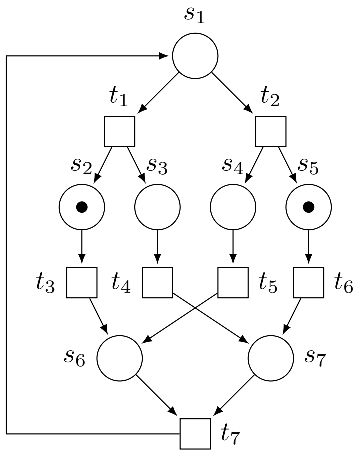
(d) Construct the constraints for the Petri net below with the markings M_0 and M.

(e) Use your constraints and the trap property to show that M is not reachable from M_0 in the net below.

Marking M_0



Marking M



Exercise 4.4 Algorithm for the largest siphon

Recall the following algorithm for computing the largest siphon Q contained in a given set R of places:

Input: A net $N = (S, T, F)$ and $R \subseteq S$.
Output: The largest siphon $Q \subseteq R$.
Initialization: $Q := R$.

begin
  while there are $s \in Q$ and $t \in {}^\bullet s$ such that $t \notin Q^\bullet$ **do**
    $Q :\ = Q \setminus \{s\}$
  endwhile
end

Show that the algorithm is correct by showing

(a) that the algorithm terminates, and

(b) that after termination, Q is the largest siphon contained in R.

Exercise 4.5 S-invariants and traps

Prove: Let N be a net and I a semi-positive S-invariant of N. The set $R = \{s \mid I(s) > 0\}$ of places is a trap of N.