Technische Universität München (I7)
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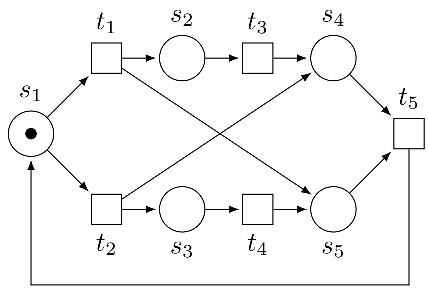
Petri nets – Homework 3

Discussed on Thursday 2nd June, 2016.

For questions regarding the exercises, please send an email to meyerphi@in.tum.de or just drop by at room 03.11.042.

Exercise 3.1 Marking equation

(a) Construct the incidence matrix $\mathbf{N}$ of the following Petri net:



(b) For the marking M marking $\{s_2, s_3\}$, solve the marking equation $M = M_0 + \mathbf{N} \cdot X$ for X. Note that $M_0 = (1, 0, 0, 0, 0)$ and $M = (0, 1, 1, 0, 0)$ as vectors with the ordering $(s_1, s_2, s_3, s_4, s_5)$ for the places. Does the equation have a solution over the integers? Does it have a solution over the natural numbers? If yes, give such a solution.

(c) Can we use the result from (b) to decide if the marking M is reachable?

Exercise 3.2 Marking equation in acyclic nets

Show the following: In a Petri net (N, M_0) which is structurally acyclic (there is no directed cycle in the net N), a marking M is reachable from M_0 *iff* there exists a nonnegative integer solution X satisfying the marking equation $M = M_0 + \mathbf{N} \cdot X$

Exercise 3.3 Transition liveness levels

For a Petri net (N, M_0) and a transition t of N, we define liveness levels in the following way:

- t is L_0-live (or dead) if t occurs in no firing sequence σ of N enabled at M_0.
- t is L_1-live if t occurs in some firing sequence σ of N enabled at M_0.
- t is L_2-live if for any $k \in \mathbb{N}$, t occurs at least k times in some firing sequence σ of N enabled at M_0.
- t is L_3-live if t occurs infinitely often in some infinite firing sequence σ of N enabled at M_0.
- t is L_4-live if for any reachable marking $M \in [M_0\rangle$, t occurs in some firing sequence σ of N enabled at M, i.e. t can always fire again. *Note*: If this holds for all transitions, this coincides with our standard definition of liveness for Petri nets.

(a) For each $i \in \{0, 1, 2, 3\}$, exhibit a Petri net (N, M_0) and a transition t of N such that t is L_i-live, but not L_{i+i}-live.

(b) For each $i \in \{0, 1, 2\}$, sketch an algorithm to decide the following problem:

Given a Petri net (N, M_0) and a transition t, is t L_i-live?

Note: You may also try to find a decision procedure for L_3-liveness, however this is non-trivial, so don't spend too much time on it.

Exercise 3.4 Number of tokens in bounded nets

Give a family of bounded Petri nets $\{N_k\}_{k \in \mathbb{N}}$ such that the size of N_k is bounded by $O(k)$ (that is, there is a $c \in \mathbb{N}$ such that for all $N_k = (S, T, F, M_0)$, we have $|S| + |T| + |F| \leq ck$ and $\forall s \in S : M_0(s) \leq ck$), but each N_k has a reachable marking M and a place s with $M(s) \geq 2^{2^k}$.

Hint: Construct a net that doubles the number of tokens in a place. Modify it so that one occurrence sequence for doubling removes exactly one token from a certain place. Use this construct again or the construct from the lecture to put 2^k tokens into that place.