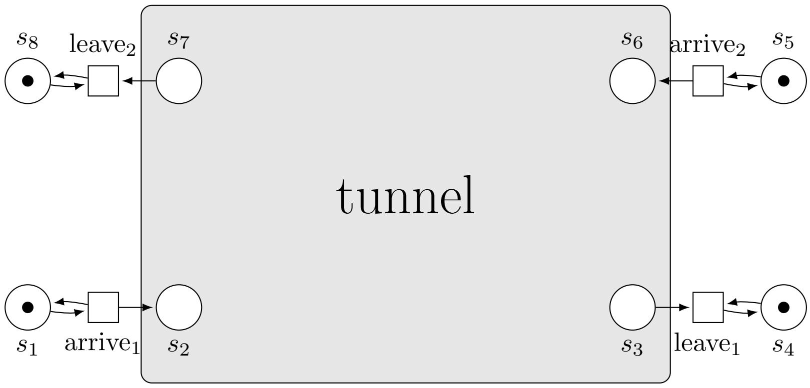
Petri nets – Homework 2

Discussed on Wednesday 18th and Thursday 19th May, 2016.

For questions regarding the exercises, please send an email to meyerphi@in.tum.de or just drop by at room 03.11.042.

Exercise 2.1 Tunnel modelling

Let us consider a tunnel connecting the TUM Campus in Garching with the Main Campus in Munich. Cars can enter the tunnel on either side and eventually exit the tunnel on the other side. A partial model of this (fictive) tunnel is given below.



(a) Extend the partial model above to a Petri net with places and transitions to keep track of the cars on each lane. Note that cars should flow from s_2 to s_3 and from s_6 to s_7. The four transitions model the environment that generates cars and absorbs cars.

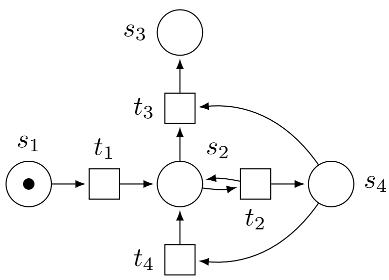
(b) The capacity of the tunnel is limited and no more than 10 cars are allowed to be in the tunnel at the same time (on both lanes together). Add this capacity constraint to your model. There should be no unnecessary waiting; for example, it cannot be the case that the tunnel remains empty while cars are queuing.

(c) As there are construction works, the tunnel is limited to one lane in one direction at time. A control system is added that can switch directions (of course only when the tunnel is empty). Modify your Petri net to incorporate this control system. Make sure that the model respects safety (no cars driving in opposite direction). However, the model does not need to be "fair".

Acknowledgment: Exercise designed by Wil van der Aalst et al. (TU Eindhoven).

Exercise 2.2 Coverability

Construct the coverability graph for the Petri net below.



Exercise 2.3 Uniqueness of the coverability graph

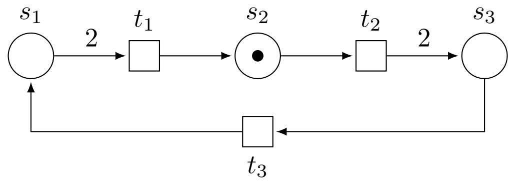
In the algorithm for the construction of the coverability graph, the search strategy (breadth-first or depth-first search and traversal order for visiting child nodes) is not specified. Show that the coverability graph obtained is not unique by exhibiting a Petri net and two different coverability graphs for this Petri net obtained by the algorithm with different search strategies.

Exercise 2.4 Reachability in Petri nets with weighted arcs

Reduce the reachability problem for Petri nets with weighted arcs to the reachability problem for Petri nets without weighted arcs.

For that, describe an algorithm that, given a Petri net with weighted arcs $N = (S, T, W, M_0)$ and a marking M, constructs a Petri net $N' = (S', T', F', M_0')$ and a marking M' such that M is reachable in N if and only if M' is reachable in N'. The algorithm should run in polynomial time (you may assume unary encoding for the weights in the input, although it is also possible with a binary encoding).

Apply the algorithm to the Petri net below with the target marking $M = (2, 0, 0)$ and give the resulting Petri net N' and marking M'.



Exercise 2.5 Monotonicity of properties

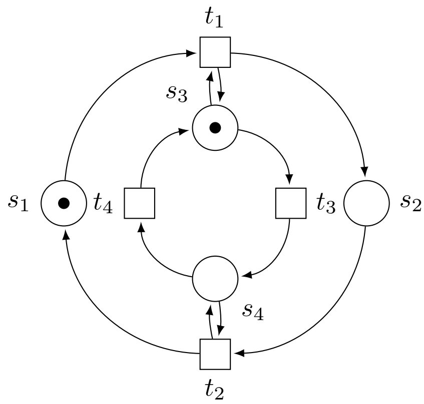
Exhibit counterexamples that disprove the following conjectures:

(a) If (N, M_0) is bounded and $M \geq M_0$, then (N, M) is bounded.

(b) If (N, M_0) is live and $M \geq M_0$, then (N, M) is live.

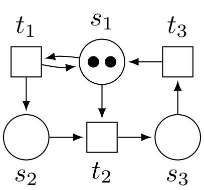
(c) If (N, M_0) is live and bounded and $M \geq M_0$, then (N, M) is bounded.

Hint: Add places and arcs to the net below to obtain a solution.



Exercise 2.6 Backwards reachability algorithm

Apply the backwards reachability algorithm to the Petri net below to decide if the marking $M = (0, 0, 2)$ can be covered. Record all intermediate sets of markings with their finite representation of minimal elements.



Exercise 2.7 Suffix sequence

Give a procedure to decide the following problem:

Given a Petri net (N, M_0) and a transition sequence σ, is there a transition sequence σ' such that $\sigma'\sigma$ is enabled at M_0?

For the procedure, you may use any already known decision procedures and algorithms such as coverability graph and backwards reachability, or you may adapt those algorithms or use parts of them.

Exercise 2.8 **Independence of boundedness, liveness and cyclicity**

Definition 2.8.1 (Cyclic Petri nets)**.** A Petri net (N, M_0) is *cyclic* if, loosely speaking, it is always possible to return to the initial marking. Formally: $\forall M \in [M_0\rangle : M_0 \in [M\rangle$.

Show that the properties liveness, boundedness and cyclicity are independent of eath other by exhibiting eight Petri nets, one for each possible combination of the three properties and their negations.

Hint: A live, bounded, but not cyclic Petri net is hard to find. A possible solution can be obtained by simply adding arcs to the net below.