Modelling concurrency with Petri nets

1. How are Petri nets a natural model for modelling concurrent processes?
2. Name some basic concurrency primitives and show how they can be modelled with Petri nets.
3. Fork/Join
4. Synchronization
5. Mutual exclusion via a semaphore
6. What is the problem when checking properties by constructing the reachability graph when you have n processes?

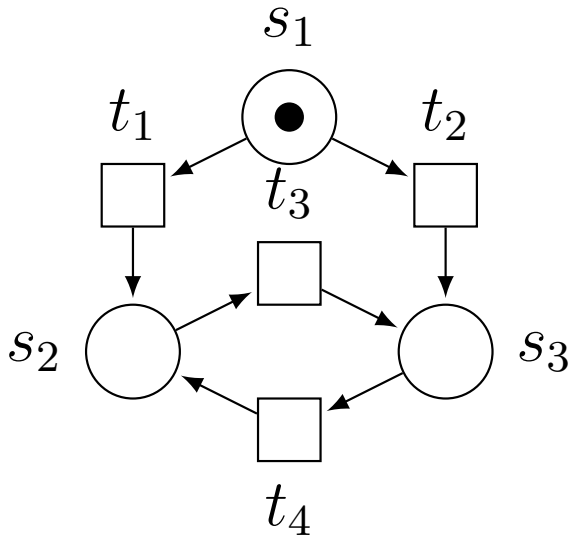
Home markings

Home marking

Let (N, M_0) be a Petri net. A marking M of the net N is a *home marking* of (N, M_0) if it is reachable from every marking of $[M_0\rangle$. We say that (N, M_0) has a home marking if some reachable marking is a home marking.

Example

This Petri net has two home markings: $(0, 1, 0)$ and $(0, 0, 1)$.



Deciding properties with the reachability graph

Boundedness

Petri net is bounded $\iff$ the reachability graph is finite.

If the net is bounded, the reachability graph can be constructed. Many properties can then be decided by analyzing the strongly connected components (SCCs) of the graph.

An SCC of the graph is a *bottom* SCC if there is no other SCC that is reachable from that SCC.

Deciding properties with a finite reachability graph

Liveness

Petri net is live $\iff$ every bottom SCC of the reachability graph contains every transition.

Deadlock freedom

Petri net is deadlock-free $\iff$ the reachability graph has no nodes with no successors.

Cyclicity

Petri net is cyclic $\iff$ the reachability graph has only one SCC.

Home marking

Petri net has a homemarking $\iff$ the reachability graph has only one bottom SCC.

Dependence between properties

- live $\Rightarrow$ deadlock-free.
- cyclic $\Rightarrow$ has a home marking.
- cyclic and not bounded $\Rightarrow$ deadlock-free

Besides these implications, the properties boundedness, liveness, deadlock freedom, cyclicity and existance of home markings are all independent of each other for unrestricted Petri nets.