Petri nets – Endterm

Last name: __________________________________________

First name: __________________________________________

Student ID no.: _______________________________________

Signature: ____________________________________________

• If you feel ill, let us know immediately.
• Please, do not write until told so.
• You will be given 90 minutes to fill in all the required information and write down your solutions.
• Don’t forget to sign.
• Write with a non-erasable pen, do not use red or green color.
• You are not allowed to use auxiliary means other than your pen and a printout of the summary.
• You may answer in English or German.
• Please turn off your cell phone.
• Should you require additional scrap paper, please tell us.
• You can obtain 40 points in the exam. You need 17 points in total to pass (grade 4.0).
• Don’t fill in the table below.
• Good luck!

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Exercise 1

Apply the backwards-reachability algorithm to the net below and the marking \( M = (0, 0, 1, 1) \) to decide if \( M \) can be covered from the initial marking \( M_0 = (1, 1, 0, 0) \). Record all intermediate steps.

![Petri net diagram]

Exercise 2

9P = 3 + 3 + 3

(a) Exhibit a live Petri net \((N, M_0)\) and a marking \( M \geq M_0 \) such that \((N, M)\) is not live. Argue succinctly why \((N, M)\) is not live (for example, by giving an occurrence sequence leading to a dead marking).

(b) Exhibit a connected net \( N \) such that \( I = (1, -1) \) is an S-invariant and \( J = (1, 1) \) is a T-invariant of \( N \). Explain your answer.

(c) Exhibit a Petri net \((N, M_0)\) and a marking \( M \) such that the marking equation \( M = M_0 + N \cdot X \) has a solution \( X : T \to \mathbb{Q} \) with \( X \geq 0 \), but no solution \( X : T \to \mathbb{N} \). Hint: A net with 2 places and 2 transitions, with no tokens in \( M_0 \) and one token in \( M \), suffices.

Exercise 3

8P = 4 + 4

(a) Exhibit a Petri net with the following reachability graph, where \( M_0 \) is the initial marking. Provide a clean drawing.

(b) Add arcs to the net below on the left such that it has the reachability graph given on the right, where \( M_0 \) is the initial marking. Provide a clean drawing.

Please turn over!
Exercise 4

For the following net, give:

(a) A positive $S$-invariant $I$ ($I$ is positive if $I(s) > 0$ for all places $s$).
(b) A positive $T$-invariant $J$ ($J$ is positive if $J(t) > 0$ for all transitions $t$).

Explain briefly the procedure you have followed to compute $I$ and $J$.

Exercise 5

A vector $X$ is semi-positive if $X \geq 0$ and $X \neq 0$.

(a) Prove: Let $N$ be a net and $I$ a semi-positive $S$-invariant of $N$. The set $R = \{s \mid I(s) > 0\}$ of places is a trap of $N$.

(b) Prove: Every live $T$-system is cyclic.

(c) Let $N = (S, T, F)$ be a net. A $T$-surinvariant of $N$ is a vector $J : T \rightarrow \mathbb{Q}$ such that $N \cdot J \geq 0$.

Prove: Let $(N, M_0)$ be a Petri net. If there is an infinite occurrence sequence $\sigma$ enabled at $M_0$, then $N$ has a semi-positive $T$-surinvariant. Hint: Use Dickson’s lemma.