Technische Universität München (I7)
Prof. J. Esparza / P. Meyer

Petri nets – Endterm

Last name: ______________________________

First name: ______________________________

Student ID no.: ______________________________

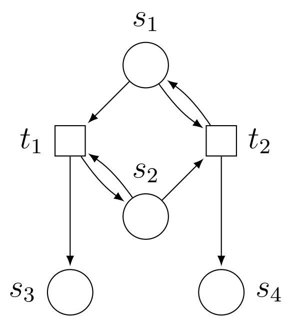
Signature: ______________________________

- If you feel ill, let us know immediately.
- Please, **do not write** until told so.
- You will be given **90 minutes** to fill in all the required information and write down your solutions.
- Don't forget to **sign**.
- Write with a non-erasable **pen**, do not use red or green color.
- You are not allowed to use **auxiliary means** other than your pen and a printout of the summary.
- You may answer in **English or German**.
- Please turn off your **cell phone**.
- Should you require additional **scrap paper**, please tell us.
- You can obtain **40 points** in the exam. You need **17 points** in total to pass (grade 4.0).
- Don't fill in the table below.
- Good luck!

Ex 1	Ex 2	Ex 3	Ex 4	Ex 5	$\sum$

Exercise 1 **6P**

Apply the backwards-reachability algorithm to the net below and the marking $M = (0, 0, 1, 1)$ to decide if M can be covered from the initial marking $M_0 = (1, 1, 0, 0)$. Record all intermediate steps.



Exercise 2 **9P=3+3+3**

(a) Exhibit a <u>live</u> Petri net (N, M_0) and a marking $M \geq M_0$ such that (N, M) is <u>not live</u>. Argue succinctly why (N, M) is not live (for example, by giving an occurrence sequence leading to a dead marking).

(b) Exhibit a connected net N such that $I = (1, -1)$ is an S-invariant and $J = (1, 1)$ is a T-invariant of N. Explain your answer.

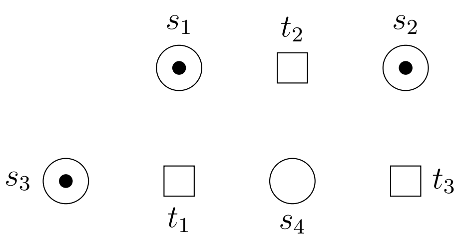
(c) Exhibit a Petri net (N, M_0) and a marking M such that the marking equation $M = M_0 + \mathbf{N} \cdot X$ has a solution $X : T \to \mathbb{Q}$ with $X \geq 0$, but no solution $X : T \to \mathbb{N}$. *Hint*: A net with 2 places and 2 transitions, with no tokens in M_0 and one token in M, suffices.

Exercise 3 **8P=4+4**

(a) Exhibit a Petri net with the following reachability graph, where M_0 is the initial marking. Provide a clean drawing.

$$
\begin{tikzcd}
 & \boxed{M_0} \arrow[r, "t_1"] & \boxed{M_1} \arrow[dr, "t_4"] & \\
\boxed{M_5} \arrow[ur, "t_4"] & & & \boxed{M_2} \arrow[dl, "t_2"] \\
 & \boxed{M_4} \arrow[ul, "t_3"] & \boxed{M_3} \arrow[l, "t_4"] &
\end{tikzcd}
$$

(b) Add arcs to the net below on the left such that it has the reachability graph given on the right, where M_0 is the initial marking. Provide a clean drawing.



$$
\begin{tikzcd}
 & & \boxed{M_2} \arrow[r, "t_1"] & \boxed{M_3} \\
\boxed{M_0} \arrow[r, "t_1"] & \boxed{M_1} \arrow[ur, "t_3"] \arrow[dr, "t_2"'] & & \\
 & & \boxed{M_4} \arrow[r, "t_3"'] & \boxed{M_5}
\end{tikzcd}
$$

Please turn over!

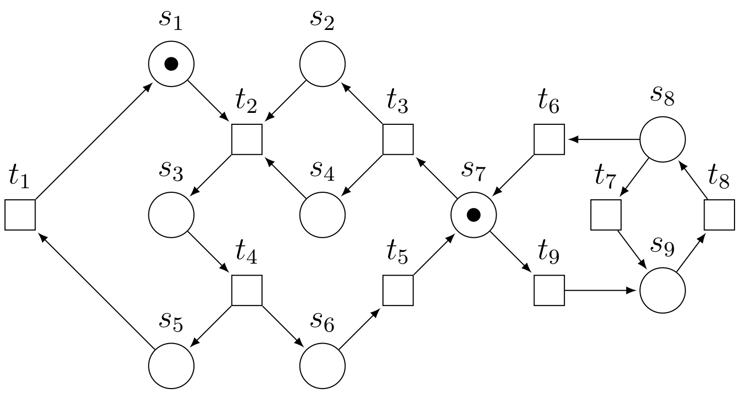
Exercise 4 **8P=4+4**

For the following net, give:

(a) A positive S-invariant I (I is positive if $I(s) > 0$ for all places s).

(b) A positive T-invariant J (J is positive if $J(t) > 0$ for all transitions t).

Explain briefly the procedure you have followed to compute I and J.



Exercise 5 **9P=3+3+3**

A vector X is *semi-positive* if $X \geq 0$ and $X \neq 0$.

(a) Prove: Let N be a net and I a semi-positive S-invariant of N. The set $R = \{s \mid I(s) > 0\}$ of places is a trap of N.

(b) Prove: Every live T-system is cyclic.

(c) Let $N = (S, T, F)$ be a net. A *T-surinvariant* of N is a vector $J : T \to \mathbb{Q}$ such that $\mathbf{N} \cdot J \geq 0$.

Prove: Let (N, M_0) be a Petri net. If there is an infinite occurrence sequence σ enabled at M_0, then N has a semi-positive T-surinvariant. *Hint*: Use Dickson's lemma.