SOLUTION

Petri nets – Resit

Last name: ______________________

First name: ______________________

Student ID no.: ______________________

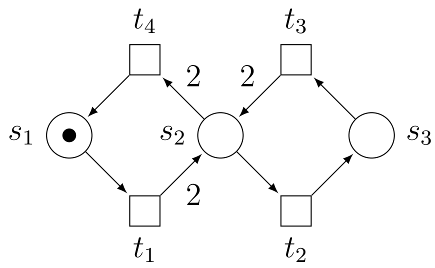
Signature: ______________________

- If you feel ill, let us know immediately.
- Please, **do not write** until told so.
- You will be given **90 minutes** to fill in all the required information and write down your solutions.
- Don't forget to **sign**.
- Write with a non-erasable **pen**, do not use red or green color.
- You are not allowed to use **auxiliary means** other than your pen and a printout of the summary.
- You may answer in **English or German**.
- Please turn off your **cell phone**.
- Should you require additional **scrap paper**, please tell us.
- You can obtain **40 points** in the exam. You need **17 points** in total to pass (grade 4.0).
- Don't fill in the table below.
- Good luck!

Ex 1	Ex 2	Ex 3	Ex 4	Ex 5	$\sum$

Exercise 1 **9P=6+3**

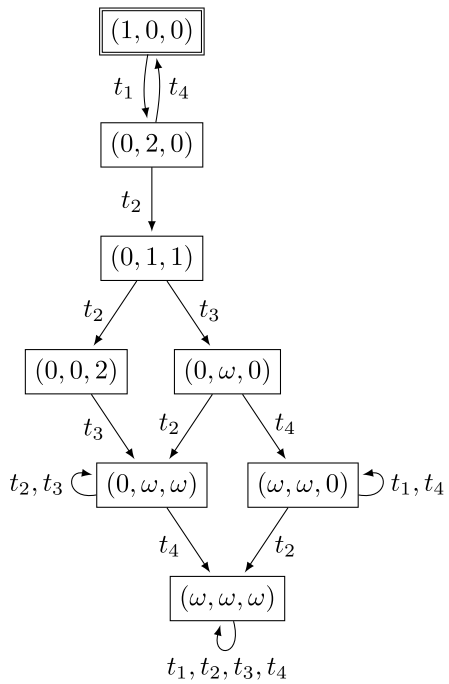
(a) Apply the coverability graph algorithm to the Petri net with weighted arcs below by using the enabling and firing rule for transitions with weighted arcs as the enabled and fire operation in the algorithm. Give the resulting coverability graph.



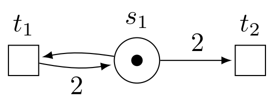
(b) Exhibit a Petri net (with or without weighted arcs) such that every node in the coverability graph of the Petri net has a successor, but the Petri net has a deadlock. Also give the coverability graph and an occurrence sequence leading to a deadlock.

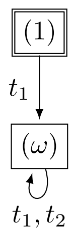
Solution:

(a) The coverability graph:

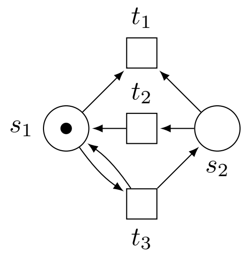


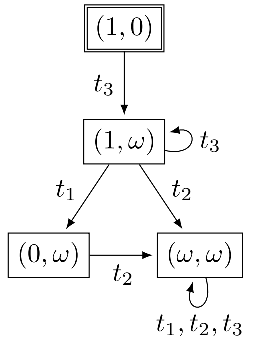
(b) A Petri net with weighted arcs with its coverability graph, where $t_1 t_2$ leads to a deadlock:





A Petri net without weighted arcs with its coverability graph, where $t_3 t_1$ leads to a deadlock:



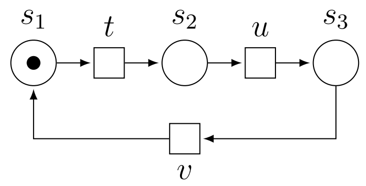


<u>**Exercise 2**</u> **8P=4+4**

For an occurrence sequence σ and a set U, we define the projection $h_U(\sigma)$ of σ onto U as the occurrence sequence obtained from σ by deleting all occurrences of transitions not in U. For example, $h_{\{t,u\}}(v) = \epsilon$, $h_{\{t,u\}}(vtv) = t$ and $h_{\{t,u\}}(tuvtuv) = tutu$.

For a Petri net (N, M_0) and a set of transitions $U \subseteq T$, we define the language of the Petri net over U as $L_U(N, M_0) = \{h_U(\sigma) \mid M_0 \xrightarrow{\sigma} M_0\}$, i.e., the set of occurrence sequences leading back to the initial marking, projected onto U.

For example, for the Petri net (N, M_0) below, we have $L_{\{t,u\}}(N, M_0) = (tu)^* = \{\epsilon, tu, tutu, tututu, \ldots\}$.



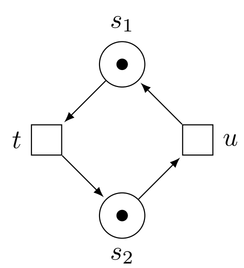
(a) Give a Petri net (N, M_0) such that $L_{\{t,u\}}(N, M_0) = (tu + ut)^*$ (another notation is $(tu \mid ut)^*$).

(b) Give a Petri net (N, M_0) such that $L_{\{t\}}(N, M_0) = (tt)^*$.

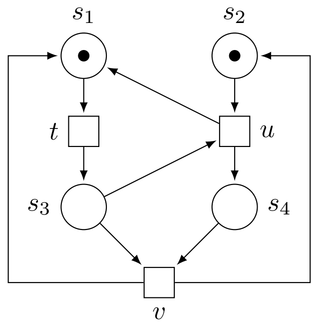
Hint: Note that a Petri net can have only one transition for each element of U.

Solution:

(a)



(b)

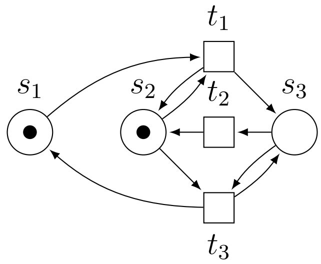


Exercise 3 **5P**

For the Petri net below, find a suitable set of S-invariants and traps, derive constraints from them over all reachable markings, and use the constraints to show that $M(s_1) < 2$ holds for all reachable markings M.

Give the S-invariants and traps, the constraints derived from them and the derivation to show $M(s_1) < 2$.

Hint: One S-invariant and one trap are sufficient.



Solution:

$I = (1, 1, 1)$ is an S-invariant, and as $I \cdot M = I \cdot M_0$, we obtain the following constraint:

$$M(s_1) + M(s_2) + M(s_3) = 2 \tag{1}$$

$R = \{s_2, s_3\}$ is a trap, and as $M_0(R) > 0$ implies $M(R) > 0$, we obtain the following constraint:

$$M(s_2) + M(s_3) > 0 \tag{2}$$

By subtracting (2) from (1), we obtain the following constraint:

$$M(s_1) < 2 \tag{3}$$

The constraint (3) shows the property.

Exercise 4 **9P=3+3+3**

A vector X is *semi-positive* if $X \geq 0$ and $X \neq 0$.

(a) Prove: Let N be a net and I a semi-positive S-invariant of N. The set $R = \{s \mid I(s) > 0\}$ of places is a siphon of N.

(b) Prove: Every live T-system has a home marking.

(c) Let $N = (S, T, F)$ be a net. A *T-surinvariant* of N is a vector $J : T \to \mathbb{Q}$ such that $\mathbf{N} \cdot J \geq 0$.

Prove: Let (N, M_0) be a Petri net. If there is an infinite occurrence sequence σ enabled at M_0, then N has a semi-positive T-surinvariant.

Solution:

(a) Let $R = \{s \mid I(s) > 0\}$. For $t \in {}^\bullet R$, there is an $s \in R$ with $s \in t^\bullet$. As $I(s) > 0$, $I(s') \geq 0$ for all $s' \in S$ and $\sum_{s \in t^\bullet} I(s) = \sum_{s' \in {}^\bullet t} I(s')$, there is an $s' \in {}^\bullet t$ with $I(s') > 0$. Therefore $s' \in R$ and $t \in R^\bullet$, so ${}^\bullet R \subseteq R^\bullet$ and R is a siphon.

(b) In a live T-system, a marking M is reachable from M_0 iff $M_0 \sim M$. Let M be a reachable marking. Then $M_0 \sim M$ and, as the relation is symmetric, $M \sim M_0$, so M_0 is reachable from M. Therefore M_0 is a home marking.

(c) Let $\sigma = t_1 t_2 t_3 \dots$ with $M_0 \xrightarrow{t_1} M_1 \xrightarrow{t_2} M_2 \xrightarrow{t_3} \dots$. By Dickson's lemma, there are indices i, j with $i < j$ and $M_i \leq M_j$. With $\sigma_1 = t_1 \dots t_i$ and $\sigma_2 = t_{i+1} \dots t_j$, we have $M_0 \xrightarrow{\sigma_1} M_i \xrightarrow{\sigma_2} M_j$. By the marking equation, we have $M_j = M_i + \mathbf{N} \cdot \vec{\sigma_2}$ and $\mathbf{N} \cdot \vec{\sigma_2} = M_j - M_i \geq 0$. As σ_2 is not empty, $\vec{\sigma_2}$ is a semi-positive T-surinvariant.

Exercise 5 — 9P=3+3+3

(a) Exhibit a Petri net (N, M_0), a trap R of N and a marking M of N such that the marking equation $M = M_0 + \mathbf{N} \cdot X$ has a solution and R is marked in M_0, but not in M.

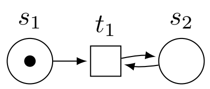
(b) Exhibit a strongly connected net N, a place s of N and an S-invariant I of N such that $I(s) > 0$, but s does not belong to any S-component.

(c) A T-invariant J of a Petri net (N, M_0) is called *realizable* if there are occurrence sequences σ and τ and a marking M such that $M_0 \xrightarrow{\sigma} M \xrightarrow{\tau} M$ and $J = \vec{\tau}$.

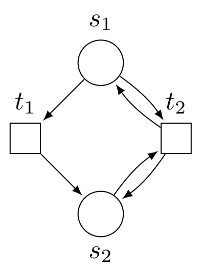
Exhibit a live Petri net (N, M_0) and a T-invariant J of N such that J is not realizable.

Solution:

(a) $R = \{s_1, s_2\}$, $M = (0, 0)$.



(b) $s = s_1$, $I = (1, 1)$.



(c) $J = (1, 0, 0, 1)$.