Petri nets – Homework 7

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For questions regarding the exercises, please send an email to meyerphi@in.tum.de or just drop by at room 03.11.042.

Exercise 7.1  Boundedness and liveness in S/T-systems

Show the following:

(a) An S-system \((N, M_0)\) is bounded for any \(M_0\).
(b) If \((N, M_0)\) is a live S-system and \(M_0' \geq M_0\), then \((N, M_0')\) is also live.
(c) If \((N, M_0)\) is a live and bounded T-system, then \((N, M_0')\) is also bounded for any \(M_0'\).
(d) If \((N, M_0)\) is a live T-system and \(M_0' \geq M_0\), then \((N, M_0')\) is also live.

Exhibit Petri nets for the following:

(e) Give a bounded T-system \((N, M_0)\) and a marking \(M_0' \geq M_0\) such that \((N, M_0')\) is not bounded.
(f) Give a 1-bounded S-system \((N, M_0)\) where \(M_0(S) > 1\).
(g) Give a live and 1-bounded T-system \((N, M_0)\) with a circuit \(\gamma\) where \(M_0(\gamma) > 1\).

Exercise 7.2  Circuits in T-systems

Consider the T-system \((N, M_0)\) below.

(a) Find all circuits of the net.
(b) Use the circuits to decide if the system is live.
(c) For each place \(s\), determine the bound of \(s\) by analyzing the circuits containing \(s\).
(d) Apply the construction from the proof of Genrich’s Theorem (Theorem 5.2.9) to find a marking \(M_0'\) such that \((N, M_0')\) is live and 1-bounded.

Exercise 7.3  Paths and transitions in T-systems

For a live T-system \((N, M_0)\), with the reachability theorem (Theorem 5.2.7), we have that a marking \(M\) is reachable iff \(M_0 \sim M\). From the proof, we can easily infer the following corollary:

Corollary 7.3.1. Let \((N, M_0)\) be a live T-system, \(M\) a marking of \(N\) and \(X : T \rightarrow N\) a vector such that \(M = M_0 + N \cdot X\). There is an occurrence sequence \(M_0 \overset{\sigma}{\rightarrow} M\) such that \(\overline{\sigma} = X\).

For a live T-system \((N, M_0)\), use this corollary to show the following. Remember that \(J = (1, \ldots, 1)\) is a T-invariant of any T-system.

(a) There is an occurrence sequence \(\sigma\) with \(M_0 \overset{\sigma}{\rightarrow} M_0\) such that \(\sigma\) contains every transition of \(N\) exactly once.
(b) For a reachable marking \(M\), there exists an occurrence sequence \(\sigma\) with \(M_0 \overset{\sigma}{\rightarrow} M\) such that \(\sigma\) does not contain all transitions of \(N\).
Exercise 7.4  
Path length in T-systems

For each \( n \in \mathbb{N} \), give a 1-bounded T-system \((\mathcal{N}, M_0)\) with \( n \) transitions and a reachable marking \( M \) such that the minimal occurrence sequence \( \sigma \) with \( M_0 \xrightarrow{\sigma} M \) has a length of \( \frac{n(n-1)}{2} \).

**Hint:** First try find a Petri net and a marking for \( n = 3 \), where the minimal sequence has length 3. For this a net with 4 places suffices. Then try to generalize your solution.