Petri nets – Homework 7

Discussed on Thursday 25^{th} June, 2015.

For questions regarding the exercises, please send an email to meyerphi@in.tum.de or just drop by at room 03.11.042.

Exercise 7.1 Boundedness and liveness in S/T-systems

Show the following:

(a) An S-system (N, M_0) is bounded for any M_0.

(b) If (N, M_0) is a live S-system and $M_0' \geq M_0$, then (N, M_0') is also live.

(c) If (N, M_0) is a live and bounded T-system, then (N, M_0') is also bounded for any M_0'.

(d) If (N, M_0) is a live T-system and $M_0' \geq M_0$, then (N, M_0') is also live.

Exhibit Petri nets for the following:

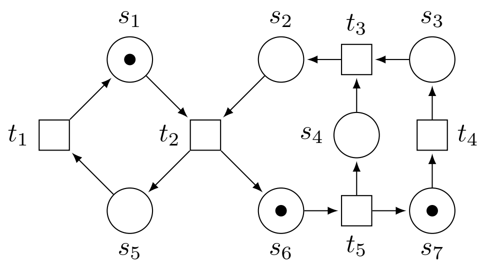
(e) Give a bounded T-system (N, M_0) and a marking $M_0' \geq M_0$ such that (N, M_0') is not bounded.

(f) Give a 1-bounded S-system (N, M_0) where $M_0(S) > 1$.

(g) Give a live and 1-bounded T-system (N, M_0) with a circuit γ where $M_0(\gamma) > 1$.

Exercise 7.2 Circuits in T-systems

Consider the T-system (N, M_0) below.



(a) Find all circuits of the net.

(b) Use the circuits to decide if the system is live.

(c) For each place s, determine the bound of s by analyzing the circuits containing s.

(d) Apply the construction from the proof of Genrich's Theorem (Theorem 5.2.9) to find a marking M_0' such that (N, M_0') is live and 1-bounded.

Exercise 7.3 Paths and transitions in T-systems

For a live T-system (N, M_0), with the reachability theorem (Theorem 5.2.7), we have that a marking M is reachable iff $M_0 \sim M$. From the proof, we can easily infer the following corollary:

Corollary 7.3.1. Let (N, M_0) be a live T-system, M a marking of N and $X : T \to \mathbb{N}$ a vector such that $M = M_0 + \mathbf{N} \cdot X$. There is an occurrence sequence $M_0 \xrightarrow{\sigma} M$ such that $\vec{\sigma} = X$.

For a live T-system (N, M_0), use this corollary to show the following. Remember that $J = (1, \ldots, 1)$ is a T-invariant of any T-system.

(a) There is an occurrence sequence σ with $M_0 \xrightarrow{\sigma} M_0$ such that σ contains every transition of N exactly once.

(b) For a reachable marking M, there exists an occurrence sequence σ with $M_0 \xrightarrow{\sigma} M$ such that σ does not contain all transitions of N.

Exercise 7.4 **Path length in T-systems**

For each $n \in \mathbb{N}$, give a 1-bounded T-system (N, M_0) with n transitions and a reachable marking M such that the minimal occurrence sequence σ with $M_0 \xrightarrow{\sigma} M$ has a length of $\frac{n(n-1)}{2}$.

Hint: First try find a Petri net and a marking for $n = 3$, where the minimal sequence has length 3. For this a net with 4 places suffices. Then try to generalize your solution.