Technische Universität München (I7)
Prof. J. Esparza / P. Meyer

Summer term 2015
Thursday 11th June, 2015

Petri nets – Homework 6

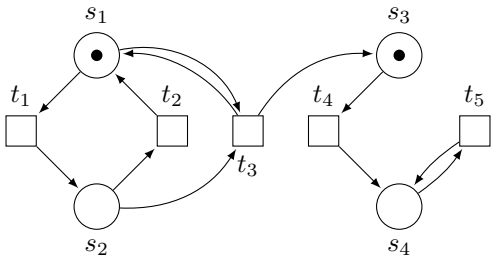
Discussed on Thursday 18th June, 2015.

For questions regarding the exercises, please send an email to meyerphi@in.tum.de or just drop by at room 03.11.042.

Exercise 6.1 Siphons and traps

(a) Find all the proper siphons and traps in the Petri net below.

(b) Check if each proper siphon contains an initially marked trap.



Exercise 6.2 Algorithm for the largest siphon

Recall the following algorithm for computing the largest siphon Q contained in a given set R of places:

Input: A net $N = (S, T, F)$ and $R \subseteq S$.
Output: The largest siphon $Q \subseteq R$.
Initialization: $Q := R$.

begin
  while there are $s \in Q$ and $t \in {}^\bullet s$ such that $t \notin Q^\bullet$ **do**
    $Q := Q \setminus \{s\}$
  endwhile
end

Show that the algorithm is correct by showing

(a) that the algorithm terminates, and

(b) that after termination, Q is the largest siphon contained in R.

Exercise 6.3 Minimal siphons

(a) Exhibit a net having a minimal siphon R and a transition t such that $|{}^\bullet t \cap R| \geq 2$.

(b) Construct for each $i \in \mathbb{N}$ a net with at most $2i$ places and at least 2^i minimal siphons.

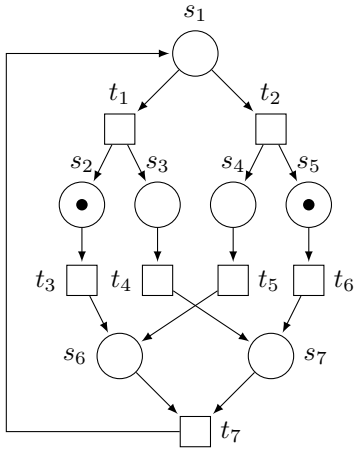
Exercise 6.4 Characterization of traps

Show the following proposition, a characterization of traps by their fundamental property.

Proposition 6.4.1. Let N be a net and R a set of places of N. R is a trap of N *iff* for all markings M of N, if $M(R) > 0$, then $M'(R) > 0$ for all $M' \in [M\rangle$.

Exercise 6.5 **Using traps to show non-reachability**

Consider the Petri net below. We want to show that M_0 is not reachable from some reachable marking M (thus showing that M_0 is not a home marking and the net is not cyclic).



(a) Find a trap R not marked at M_0.

(b) Find a marking M reachable from M_0 that marks R.

(c) Use R to construct a constraint over the markings reachable from M and show that M_0 is not reachable from M.

Exercise 6.6 **Linear inequation net**

Consider the following set, defined by a linear inequation.

$$X = \{(x_1, x_2, x_3, x_4) \in \mathbb{N}^4 \mid 2x_1 + 5x_2 \leq 3x_3 + 4x_4\}$$

Give a Petri net (N, M_0) (with or without weighted arcs) containing four designated places x_1, x_2, x_3 and x_4 (and possibly other places) such that $\{(M(x_1), M(x_2), M(x_3), M(x_4)) \mid M \in [M_0\rangle\} = X$, i.e., the reachable markings represent the set X.