Technische Universität München (I7)
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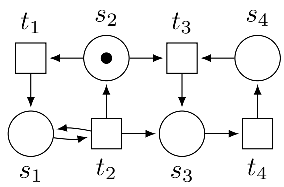
Petri nets – Homework 5

Discussed on Thursday 11th June, 2015.

For questions regarding the exercises, please send an email to meyerphi@in.tum.de or just drop by at room 03.11.042.

Exercise 5.1 Marking equation

(a) Construct the incidence matrix **N** of the following Petri net:



(b) Use the marking equation to decide whether the following markings are not reachable, or may be reachable. Does it make a difference whether the solution space is restricted to the natural numbers or to the rationals?

$$M_1 = (0,0,0,0) \qquad M_2 = (1,1,1,1) \qquad M_3 = (1,4,1,1)$$

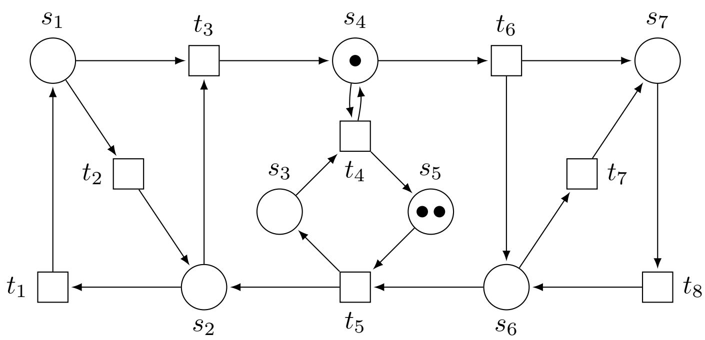
Exercise 5.2 Marking equation in acyclic nets

Show the following: In a Petri net (N, M_0) which is structurally acyclic (there is no directed cycle in the net N), a marking M is reachable from M_0 *iff* there exists a nonnegative integer solution X satisfying the marking equation $M = M_0 + \mathbf{N} \cdot X$

Exercise 5.3 S-invariants and T-invariants

Give a basis of the space of S-invariants and a basis of the space of T-invariants of the following net. Does the net have positive S-invariants and T-invariants? Can you make any statements about the boundedness and liveness of the net based on the invariants?

Hint: Use the alternative definitions for S-invariants and T-invariants to find them more easily.



Exercise 5.4 Bounded net with no positive S-invariant

(a) Exhibit a Petri net (N, M_0) which is bounded, but has no positive S-invariant.

(b) As (a), but (N, M_0) is required to be live and bounded.

Exercise 5.5 Reproduction lemma

Let (N, M_0) be a bounded system and let $M_0 \xrightarrow{\sigma}$ be an infinite occurrence sequence. Show the following:

(a) There exists sequences σ_1, σ_2, σ_3 such that $\sigma = \sigma_1\sigma_2\sigma_3$, σ_2 is not the empty sequence and

$$M_0 \xrightarrow{\sigma_1} M \xrightarrow{\sigma_2} M \xrightarrow{\sigma_3}$$

for some marking M.

(b) There exists a semi-positive T-invariant J such that $\langle J \rangle \subseteq \mathcal{A}(\sigma)$, where $\mathcal{A}(\sigma)$ is the set of transitions appearing in σ.