Technische Universität München (I7)
Prof. J. Esparza / P. Meyer

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Petri nets – Homework 4

Discussed on Thursday 28th May, 2015.

For questions regarding the exercises, please send an email to meyerphi@in.tum.de or just drop by at room 03.11.042.

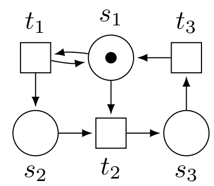
Exercise 4.1 Coverability with transfer arcs

In exercise 3.4, we defined Petri nets with transfer arcs and showed that coverability is decidable with the backwards reachability algorithm. However, this does not work with the coverability graph algorithm, as spurios ω may be introduced.

Give a Petri net with transfer arcs where there are occurrence sequences $M_0 \xrightarrow{\sigma_0} M_1 \xrightarrow{\sigma_1} M_2$ such that $M_2 \geq M_1$ and $M_2 \neq M_1$ (thus introducing an ω in the coverability graph), but the net is bounded.

Exercise 4.2 Backwards reachability algorithm

Apply the backwards reachability algorithm to the Petri net below to decide if the marking $M = (0, 0, 2)$ can be covered. Record all intermediate sets of markings with their finite representation of minimal elements.

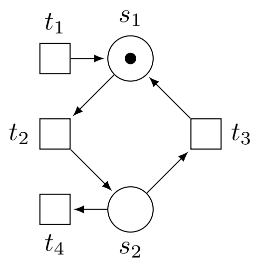


Exercise 4.3 Reduction of reachability problems

A variant of the reachability problem is the *zero-reachability problem*:

Definition 4.3.1 (Zero-reachability problem)**.** For a Petri net (N, M_0), is there a marking $M \in [M_0\rangle$ with $M(s) = 0$ for all $s \in S$?

Let $\mathbf{0}$ be the marking M with $M(s) = 0$ for all $s \in S$. A reduction from the zero-reachability problem to the reachability problem is straightforward: simpliy specify the target marking $\mathbf{0}$. For the other way, reduce the reachability problem to the zero-reachability problem. Describe an algorithm that, given a Petri net (N, M_0) and a marking M, constructs in polynomial time a Petri net (N', M_0') such that M is reachable from M_0 in N if and only if $\mathbf{0}$ is reachable from M_0' in N'. Apply the algorithm to the Petri net below with the marking $M = (0, 2)$.

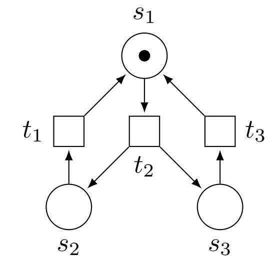


Exercise 4.4 Semilinear sets

(a) Show that the following set is semilinear by giving a finite set of pairs of roots and periods $\{(r_1, P_1), \ldots, (r_n, P_n)\}$ representing the linear sets.

$$X = \{(x_1, x_2, x_3) \in \mathbb{N}^3 \mid x_1 \leq x_2 + 1 \leq x_3\}$$

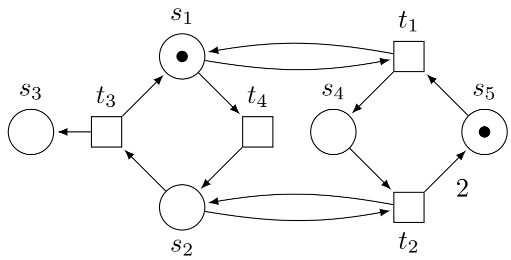
(b) Show that the set of reachable markings of the following Petri net is semilinear.



(c) Use the representation of the reachable markings of the previous Petri net as a semilinear set to show that the marking $M = (0, 0, 1)$ is not reachable.

Exercise 4.5 Petri net with a non-semilinear reachability set

We want to show that the following Petri net with weighted arcs has a non-semilinear reachability set (we can also obtain the result with unweighted arcs by the standard reduction).



Consider the following sets of markings, given as $M = (s_1, s_2, s_3, s_4, s_5)$:

$$\mathcal{M}_1 = \{(1, 0, x_1, x_2, x_3) \mid 0 < x_2 + x_3 \leq 2^{x_1}\}$$
$$\mathcal{M}_2 = \{(0, 1, x_1, x_2, x_3) \mid 0 < 2x_2 + x_3 \leq 2^{x_1+1}\}$$
$$\mathcal{M} = \mathcal{M}_1 \cup \mathcal{M}_2$$

Clearly, $\mathcal{M}$ is a non-semilinear set. We claim: $\mathcal{M}$ is equal to the set of reachable markings for the above Petri net.

(a) Show that if $M \in [M_0\rangle$, then $M \in \mathcal{M}$. For this, show that $M_0 \in \mathcal{M}$ and if $M \in \mathcal{M}$ and $M \xrightarrow{t} M'$ for some transition t, then also $M' \in \mathcal{M}$.

(b) Show that if $M \in \mathcal{M}$, then $M \in [M_0\rangle$.

Note: This is a rather hard exercise. *Hint*: Do this by induction on $x_1 = M(s_3)$ for $M \in \mathcal{M}$. In the induction step at x_1, do a case distinction between $M \in \mathcal{M}_1$ and $M \in \mathcal{M}_2$. In each case, find an M' for which you can apply the induction hypothesis and from which M is reachable.