Technische Universität München (I7)
Prof. J. Esparza / P. Meyer

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Petri nets – Homework 3

Discussed on Thursday 21st May, 2015.

For questions regarding the exercises, please send an email to meyerphi@in.tum.de or just drop by at room 03.11.042.

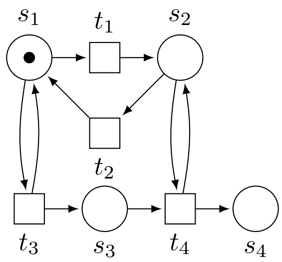
Exercise 3.1 Coverability

Construct the coverability graph for the Petri net below.

(a) List the unbounded places of the Petri net.

(b) Decide if the following markings $M = (s_1, s_2, s_3, s_4)$ are coverable:

$$M_1 = (1,0,0,0) \quad M_2 = (0,0,1,0) \quad M_3 = (1,1,0,0) \quad M_4 = (0,0,1,1) \quad M_5 = (1,0,1,0) \quad M_6 = (0,1,0,1)$$



(a) The places s_3 and s_4 are unbounded, as there are ω-markings M, M' with $M(s_3) = \omega$ and $M'(s_4) = \omega$.

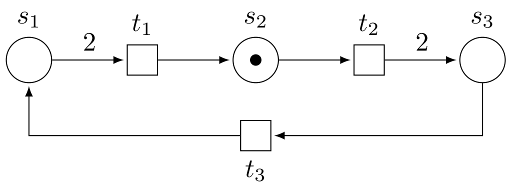
(b) The markings M_1, M_2, M_4 and M_5 are covered by $(1, 0, \omega, \omega)$ and M_6 is covered by $(0, 1, \omega, \omega)$. The marking M_3 is not coverable, as there is no ω-marking M with $M(s_1) \geq 1$ and $M(s_2) \geq 1$.

Exercise 3.2 Reachability in Petri nets with weighted arcs

Reduce the reachability problem for Petri nets with weighted arcs to the reachability problem for Petri nets without weighted arcs.

For that, describe an algorithm that, given a Petri net with weighted arcs $N = (S, T, W, M_0)$ and a marking M, constructs a Petri net $N' = (S', T', F', M_0')$ and a marking M' such that M is reachable in N if and only if M' is reachable in N'. The algorithm should run in polynomial time (you may assume unary encoding for the weights in the input, although it is also possible with a binary encoding).

Apply the algorithm to the Petri net below with the target marking $M = (2, 0, 0)$ and give the resulting Petri net N' and marking M'.



Exercise 3.3 Uniqueness of the coverability graph

In the algorithm for the construction of the coverability graph, the search strategy (breadth-first or depth-first search and traversal order for visiting child nodes) is not specified. Show that the coverability graph obtained is not unique by exhibiting a Petri net and two different coverability graphs for this Petri net obtained by the algorithm with different search strategies.

Exercise 3.4 Backwards reachability with transfer arcs

Another variant of Petri nets are nets with transfer arcs, a generalization of nets with reset arcs:

Definition 3.4.1 (Nets with transfer arcs)**.** A *net with transfer arcs* $N = (S, T, F, R)$ consists of two disjoint sets of places and transitions, a set $F \subseteq (S \times T) \cup (T \times S)$ of arcs, and a set $R \subseteq (S \times T) \cup (T \times S)$, disjoint from F, of *transfer arcs.*

A transition t is *enabled* at a marking M of N if $M(s) > 0$ for every place s such that $(s, t) \in F \cup R$. If t is enabled then it can *occur* leading to the marking M' obtained after the following operations:

1. Let k be the sum of the tokens in all places s such that $(s, t) \in R$, i.e., $k := \sum_{\{s \in S | (s,t) \in R\}} M(s)$.
2. Remove one token from every place s such that $(s, t) \in F$.
3. Remove all tokens from every place s such that $(s, t) \in R$.
4. Add one token to every place s such that $(t, s) \in F$.
5. Add k tokens to every place s such that $(t, s) \in R$.

Show that the abstract backwards-reachability algorithm can be applied to Petri nets with transfer arcs by showing that the transition relation is monotonic.

Exercise 3.5 Number of tokens in bounded nets

Give a family of bounded Petri nets $\{N_k\}_{k \in \mathbb{N}}$ such that the size of N_k is bounded by $O(k)$ (that is, there is a $c \in \mathbb{N}$ such that for all $N_k = (S, T, F, M_0)$, we have $|S| + |T| + |F| \leq ck$ and $\forall s \in S : M_0(s) \leq ck$), but each N_k has a reachable marking M and a place s with $M(s) \geq 2^{2^k}$.

Hint: Construct a net that doubles the number of tokens in a place. Modify it so that one occurrence sequence for doubling removes exactly one token from a certain place. Use this construct again or the construct from the lecture to put 2^k tokens into that place.