**Technische Universität München (I7)**
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# Petri nets – Homework 2

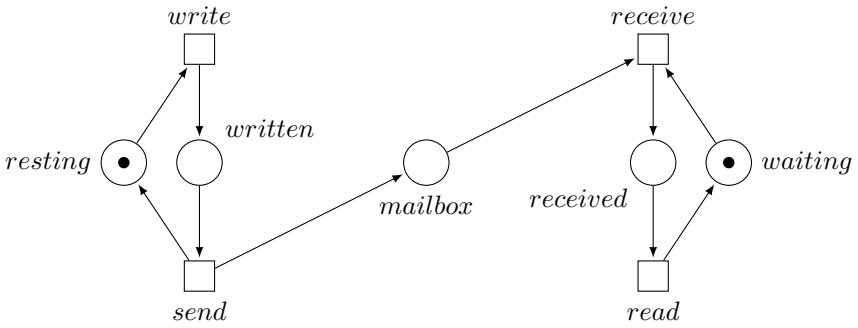
Discussed on Wednesday 13th May, 2015.

*For questions regarding the exercises, please send an email to meyerphi@in.tum.de or just drop by at room 03.11.042.*

## Exercise 2.1 Bounded mailbox

The following Petri net models a communication of two processes by sending mails to a mailbox and receiving them from the mailbox. This is an instance of the producer/consumer problem.

Currently the mailbox is unbounded. Show how, for a given $k$, the net can be modified to enforce a maximum capacity of $k$ on the mailbox, that is, the place *mailbox* should be $k$-bounded in the modified net. Try to minimize the modifications.




## Exercise 2.2 A vending machine

We would like to model a simple vending machine with a Petri net. The vending machine accepts 1 euro coins and dispenses chocolate bars for 1 euro each. After inserting 1 euro, the machine should eventually dispense a chocolate bar. A chocolate bar should only be dispensed after inserting one euro, and one euro is only valid for one chocolate bar.

(a) Model the vending machine as a Petri net. It should have transitions for inserting a 1 euro coin and for dispensing a chocolate bar.

(b) Modify the Petri net so that the vending machine has a limited amount of storage for chocolate bars, and initially contains 4 bars. The net should have a transition for refilling the storage, however not above the maximum capacity of 4 bars.

(c) Now the vending machine is upgraded to offer both small chocolate bars for 1 euro and large chocolate bars for 2 euros. Modify the Petri net such that it additionally allows for dispension of large chocolate bars after inserting at least 2 euros. The large chocolate bars are in a separate storage space, which should be limited to 3 bars and also be able to be refilled.

*Note:* You may use Petri nets with weighted arcs. However, also think about a solution without weights. For this, you may have more than one transition corresponding to the insertion of a coin or to the dispension of a chocolate bar.

(d) Further modify the Petri net so that the vending machine also accepts 2 euro coins. A 2 euro coin should allow dispension of either two small chocolate bars or one large chocolate bar.

## Exercise 2.3 Monotonicity of properties

Exhibit counterexamples that disprove the following conjectures:

(a) If $(N, M_0)$ is bounded and $M \geq M_0$, then $(N, M)$ is bounded.

(b) If $(N, M_0)$ is live and $M \geq M_0$, then $(N, M)$ is live.

(c) If $(N, M_0)$ is live and bounded and $M \geq M_0$, then $(N, M)$ is bounded.

## Exercise 2.4 Home markings

**Definition 2.4.1** (Home marking)**.** Let $(N, M_0)$ be a Petri net. A marking $M$ of the net $N$ is a *home marking* of $(N, M_0)$ if it is reachable from every marking of $[M_0\rangle$.

We say that $(N, M_0)$ has a home marking if some reachable marking is a home marking.

(a) Does the Petri net from exercise 1.2 have a home marking?

(b) Find all home markings for the three Petri nets $A$, $B$ and $C$ from exercise 1.4.

(c) Exhibit a Petri net $(N, M_0)$ which has home markings, but also an infinite occurrence sequence $M_0 \xrightarrow{\sigma}$ such that none of the markings along the occurrence of $\sigma$ is a home marking.

(d) Show that every reachable marking of a cyclic Petri net is a home marking.

(e) Prove that every bounded Petri net $(N, M_0)$ has a reachable marking $M$ which is a home marking of $(N, M)$.

## Exercise 2.5* Live and bounded Petri net without home markings

*Note:* This is a bonus exercise, as it is rather challenging. Only do it if you are interested and have enough time.

Exhibit a live and bounded Petri net without home markings.