Technische Universität München (I7) **Summer 2013**
Javier Esparza

Petri Nets – Endterm

Last name: ______________________________

First name: ______________________________

Student ID no.: ______________________________

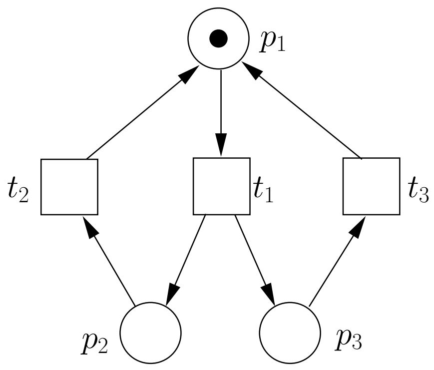
Signature: ______________________________

- If you feel ill, let us know immediately.
- You have **75 minutes** to fill in all the required information and write down your solutions.
- Don't forget to **sign**.
- Write with a non-erasable **pen**, do not use red or green color.
- You are not allowed to use **auxiliary means**.
- You may answer in **English or German**.
- Please turn off your **cell phone**.
- Should you require additional **scrap paper**, please tell us.
- You can obtain **40 points** in the exam. You need **17 points** in total to pass (grade 4.0).
- Don't fill in the table below.
- Good luck!

Ex1	Ex2	Ex3	Ex4	$\sum$

Exercise 1 **6P**

Construct the coverability graph of the Petri net



Exercise 2 **14P=4+4+3+3**

(a) Exhibit a net such that the vector $(1, 1, 2)$ is a place invariant (P-invariant, S-invariant) and the vector $(1, 1)$ is a transition invariant (T-invariant). Show that this is indeed the case.

(b) Exhibit a live net without place invariants or transition invariants (apart from the trivial ones in which all components are equal to 0). Show that this is indeed the case.

(c) Exhibit four T-systems (every place has exactly one input and one output transition) that are, respectively: live and bounded, live and unbounded, non-live and bounded, non-live and unbounded. Say which system satisfies which properties.

(e) Prove or disprove: If (M, M_0) is a live free-choice Petri net and $M \geq M_0$ (i.e., $M(p) \geq M_0(p)$ for every place p), then (N, M) is also live.

Exercise 3 **10P=3+3+4**

We consider nets with four distinguished places called `Start`, `Input_1`, `Input_2`, and `Result`. The nets may also have other places.

Given such a net N, we denote by $[n_1, n_2]$ the marking that puts:

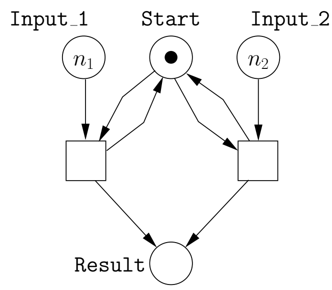
1 token in `Start` n_1 tokens in `Input_1` n_2 tokens in `Input_2` 0 tokens elsewhere.

We say that N **weakly computes** a function $f(x_1, x_2)$ if for every $n_1, n_2 \in \mathbb{N}$ (where $\mathbb{N}$ contains 0) the following two conditions hold:

- **some** reachable marking of N with $[n_1, n_2]$ as initial marking puts $f(n_1, n_2)$ tokens in `Result`; and
- **no** reachable marking of N with $[n_1, n_2]$ as initial marking puts more than $f(n_1, n_2)$ tokens in `Result`.

We do not care about the number of tokens in other places.

For instance, the Petri net below weakly computes the function $f(x_1, x_2) = x_1 + x_2$.



Give Petri nets that weakly compute

(a) the function $f(x_1, x_2) = \min\{x_1, x_2\}$;

(b) the function $f(x_1, x_2) = \max\{x_1, x_2\}$;

(c) the function $f(x_1, x_2) = x_1 \cdot x_2$.

In all cases 6 places suffice.

Please turn over!

Exercise 4 **10P=5+2+2+1**

Reduce the satisfiability problem for boolean formulas in conjunctive normal form to the following problem:

> **Given:** an acyclic Petri net.
> **Decide:** is the empty marking (no tokens anywhere) reachable?

Recall that a reduction is a polynomial time translation from boolean formulas into acyclic Petri nets such that a formula is satisfiable iff in its Petri net translation the empty marking is reachable. You do not have to describe the translation formally, but you have to:

(a) Draw the result of applying the translation to the formula

$$(x_1 \vee x_2 \vee \neg x_3) \wedge (\neg x_1 \vee x_3)$$

(b) Give a brief argument showing that if the formula is satisfiable, then the empty marking is reachable.

(c) Give a brief argument showing that if the formula is not satisfiable, then the empty marking is not reachable.

(d) Give a brief argument showing that the translation takes polynomial time.