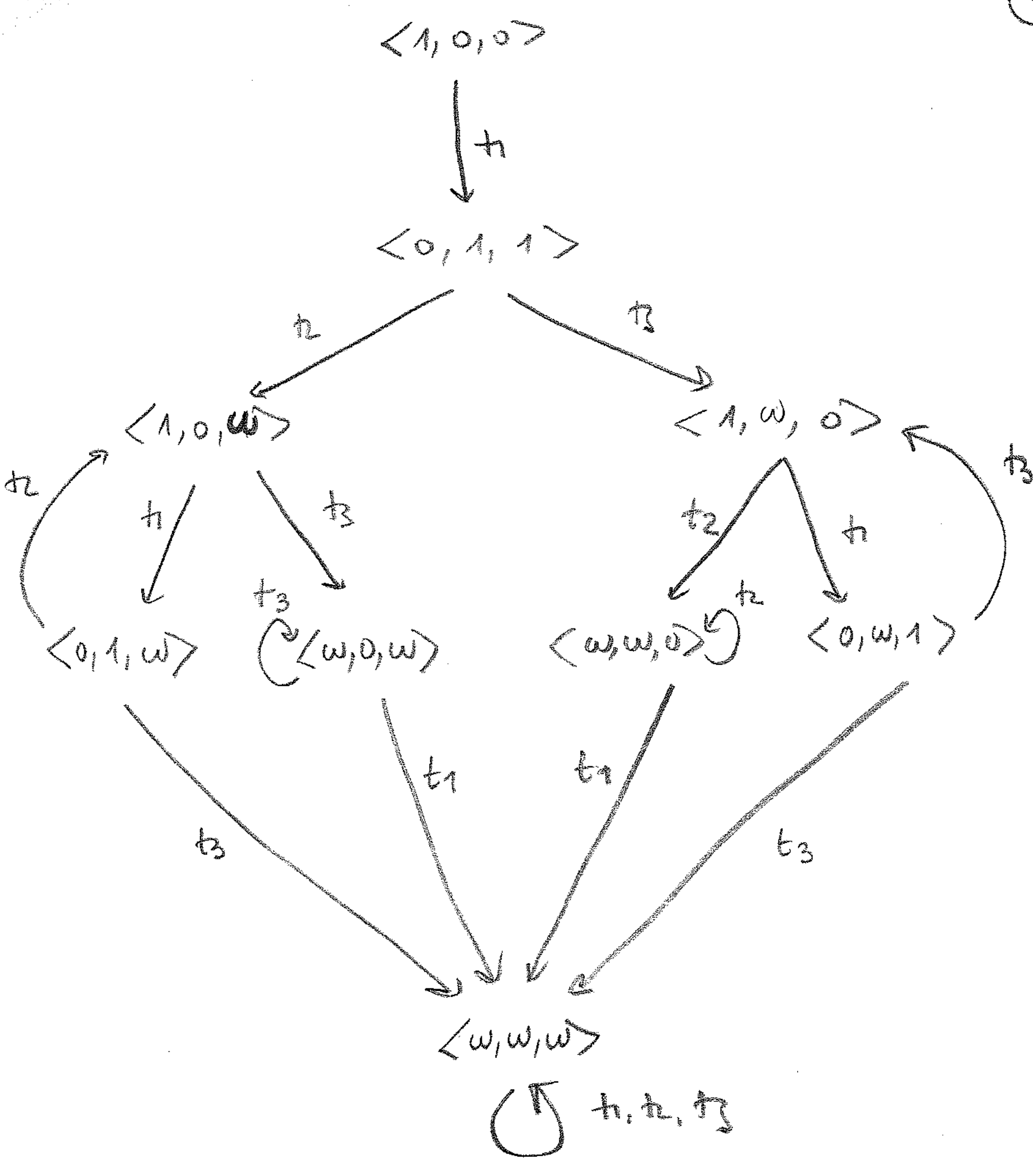
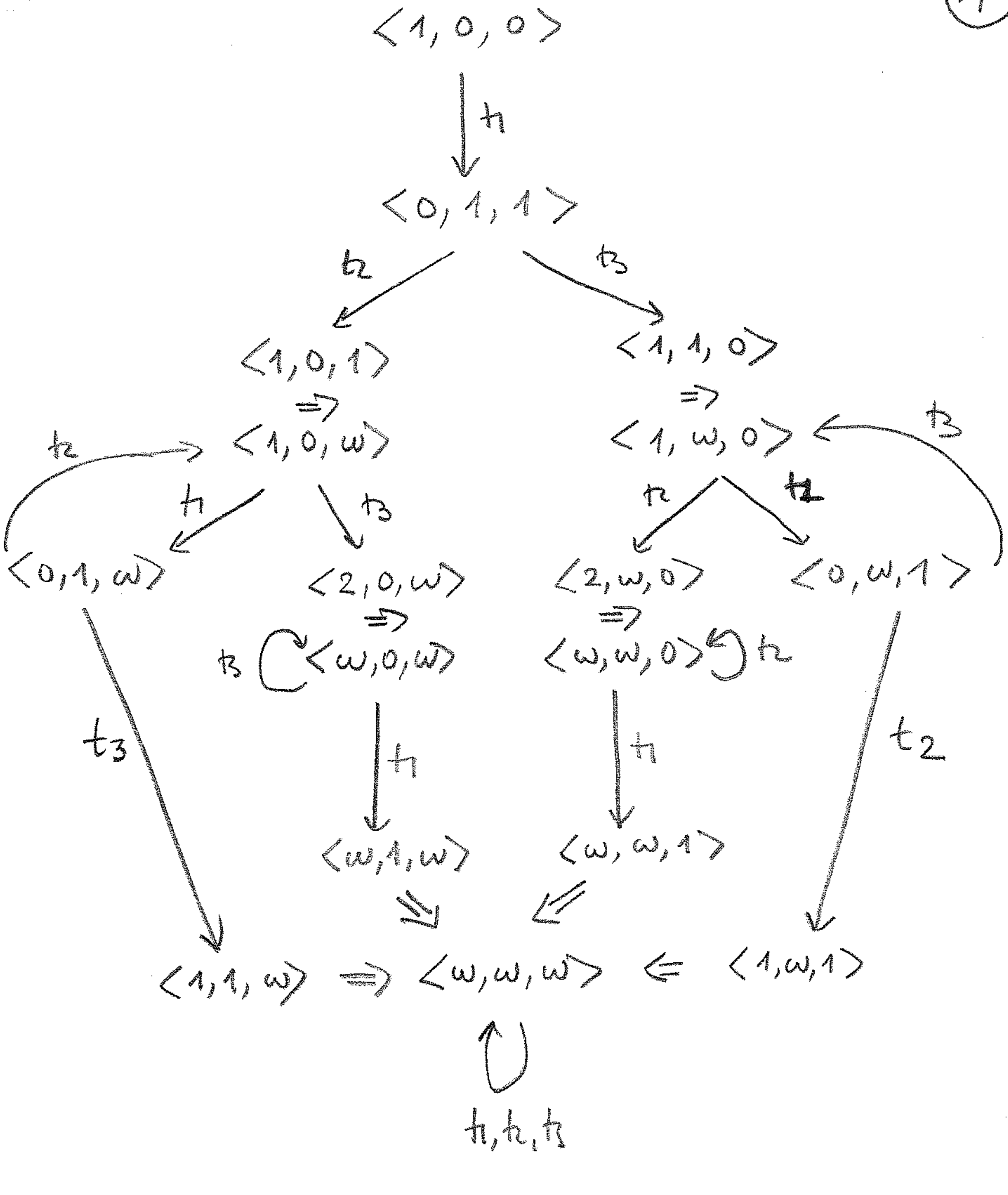
Petri nets

SS 2013

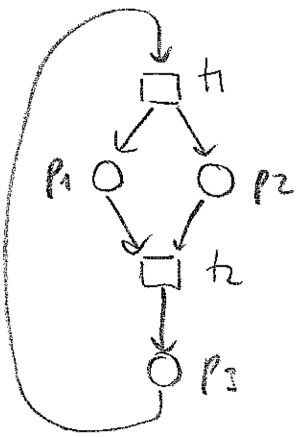
Solution to the exam


⟨1, 0, 0⟩
t1
⟨0, 1, 1⟩
t2
t3
⟨1, 0, ω⟩
⟨1, ω, 0⟩
t2
t1
t3
t3
t2
t1
t2
⟨0, 1, ω⟩
t3
⟨ω, 0, ω⟩
⟨ω, ω, 0⟩
t2
⟨0, ω, 1⟩
t3
t1
t1
t3
⟨ω, ω, ω⟩
t1, t2, t3



(includes the addition of ω's)

(a)



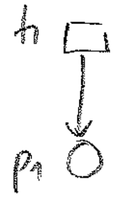
Incidence matrix :

$$C = \begin{array}{cc} t_1 & t_2 \end{array} \atop \left(\begin{array}{cc} 1 & -1 \\ 1 & -1 \\ -1 & 1 \end{array}\right) \begin{array}{c} p_1 \\ p_2 \\ p_3 \end{array}$$

$$C \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 0 \quad \Rightarrow \quad (1,1) \text{ is T-invariant}$$

$$(1\ 1\ 2) \cdot C = 0 \quad \Rightarrow \quad (1,1,2) \text{ is P-invariant.}$$

(b)

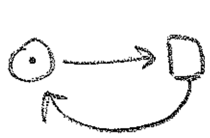


Incidence matrix :

$$C = \begin{array}{c} t_1 \\ (\ 1\ ) \end{array} \ p_1$$

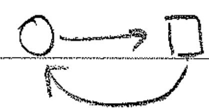
There are no vectors $X, Y \neq 0$ such that $C \cdot X = 0$, $Y \cdot C = 0$ $\Rightarrow$ the net has no T-invariant and no P-invariant.

(c)

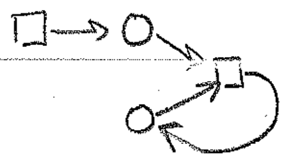


live and bounded

live and unbounded



non-live and bounded

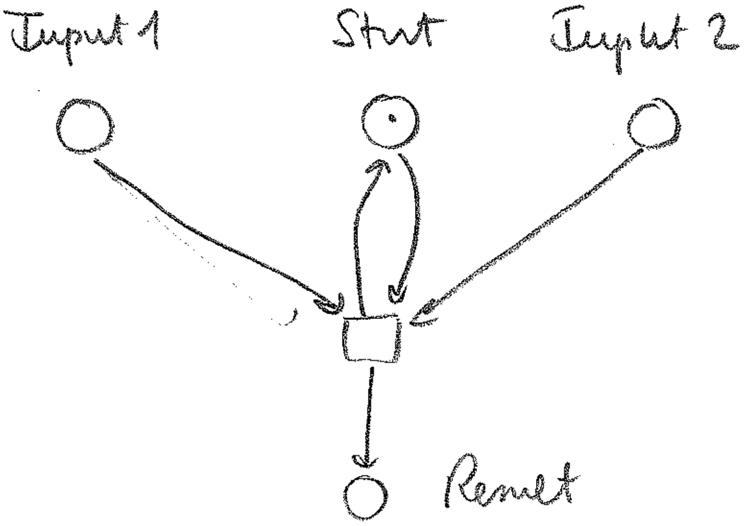


non-live and unbounded.

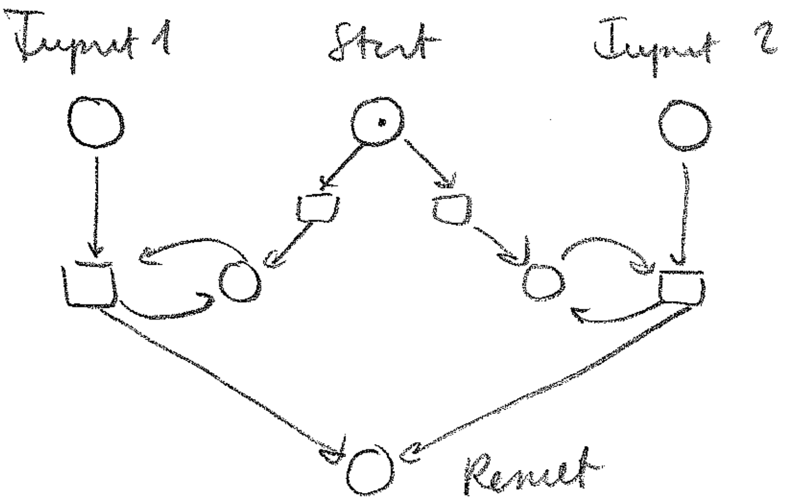
By Commoner's theorem, a free-choice Petri net is live if and only if every proper siphon contains an initially marked trap.

Assume a free-choice net (N, M_0) is live and $M \geq M_0$.
By Commoner's theorem, every proper siphon of N contains a trap marked at M_0.
Since $M \geq M_0$, every proper siphon of N contains a trap marked at M.
By Commoner's theorem, (N, M) is live.

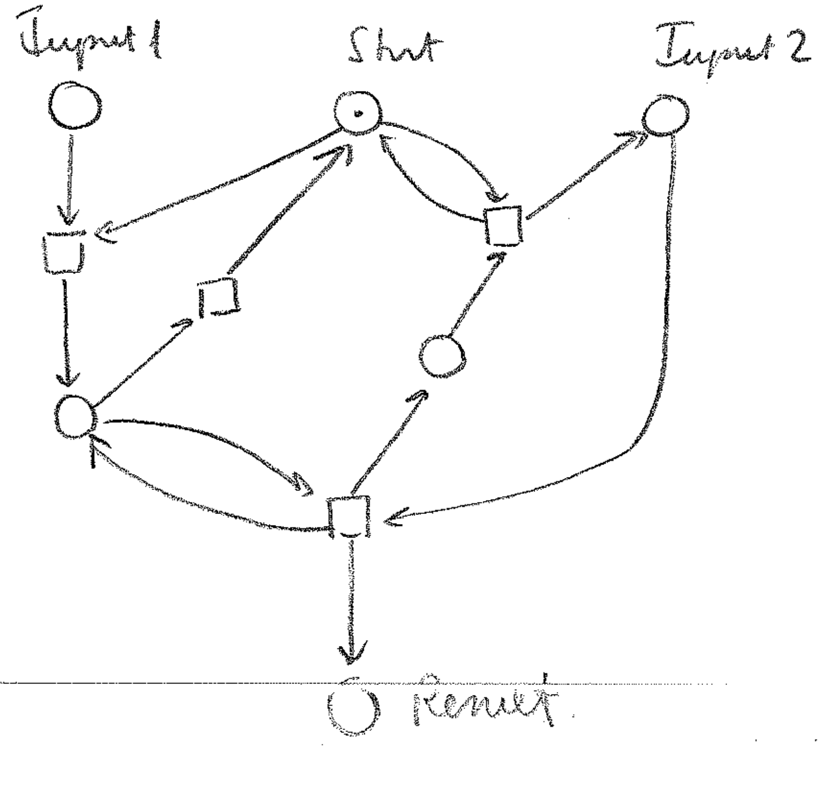
a)

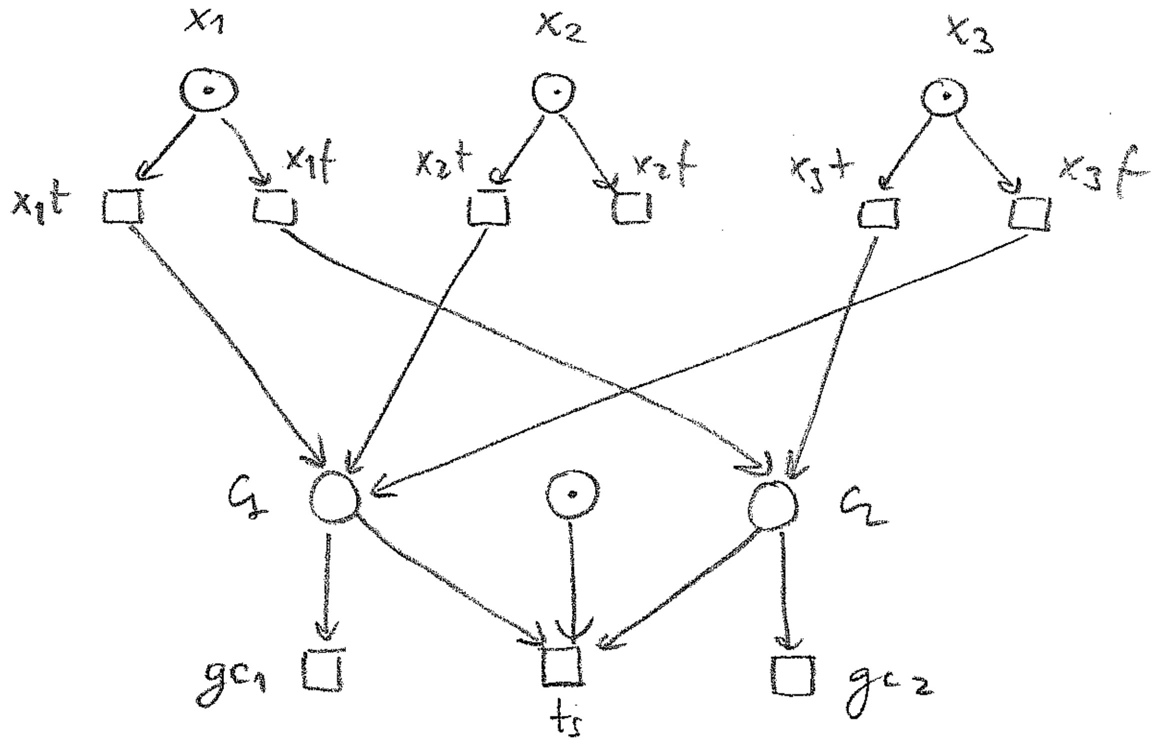

Input 1
Start
Input 2
Result

b)


Input 1
Start
Input 2
Result

c)


Input 1
Start
Input 2
Result



b) If the formula is satisfiable, then there is a truth assignment that satisfies all clauses. Firing the transitions corresponding to this assignment puts at least one token (possibly up to three) in each of the places C_i. This allows to fire transition t_s, and the transitions $gc_1, \ldots, gc_m$ can take all remaining tokens from $c_1, \ldots, c_m$, reaching the empty marking

c) If the formula is unsatisfiable, then independently of how the transitions at the top occur, after one of each pair has fired at least one of the places $C_1, \ldots, C_m$ is not marked. So the transition t_s can never occur, and so the empty marking is not reachable

The number of places is linear in the size of the formula, and so is the number of transitions (number of places is bounded by $n+m+1$ for a formula with n variables and m clauses, and number of transitions is bounded by $2n+m+1$). Moreover, the places and transitions can be constructed in linear time during one single scan of the formula.