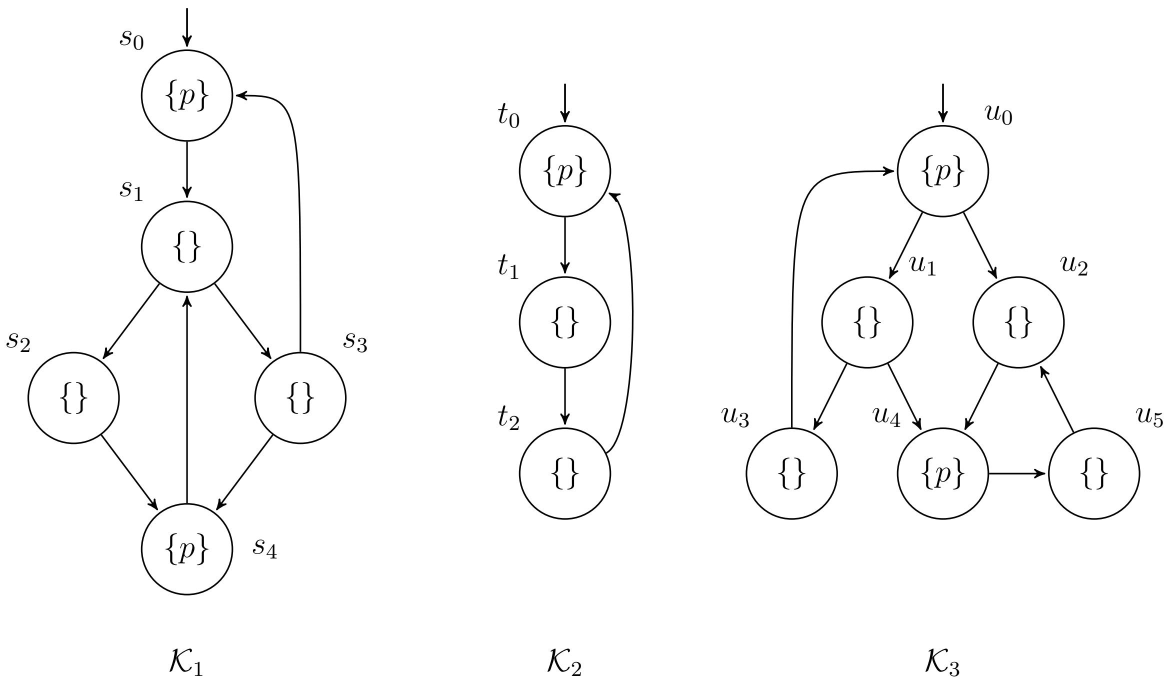
Model Checking – Exercise sheet 10

Exercise 10.1
Consider the following Kripke structures $\mathcal{K}_1$, $\mathcal{K}_2$, and $\mathcal{K}_3$, over $AP = \{p\}$:



(a) Does $\mathcal{K}_2$ simulate $\mathcal{K}_1$? If yes, give a simulation relation. Otherwise, explain why.

(b) Does $\mathcal{K}_2$ simulate $\mathcal{K}_3$? If yes, give a simulation relation. Otherwise, explain why.

(c) Does $\mathcal{K}_3$ simulate $\mathcal{K}_2$? If yes, give a simulation relation. Otherwise, explain why.

(d) Does $\mathcal{K}_3$ simulate $\mathcal{K}_1$? If yes, give a simulation relation. Otherwise, explain why.

Exercise 10.2
Let $\mathcal{K}_1$, $\mathcal{K}_2$, and $\mathcal{K}_3$ be Kripke structures. Show that if $\mathcal{K}_1$ and $\mathcal{K}_2$ are bisimilar, and $\mathcal{K}_2$ and $\mathcal{K}_3$ are bisimilar, then $\mathcal{K}_1$ and $\mathcal{K}_3$ are also bisimilar.

Exercise 10.3
(Taken from 'Principles of Model Checking')
Let $TS = (S, Act, \rightarrow, I, AP, L)$ be a transition system. A bisimulation for TS is a binary relation R on S such that for all $(s_1, s_2) \in R$:

- $L(s_1) = L(s_2)$.
- If $s_1' \in Post(s_1)$, then there exists an $s_2' \in Post(s_2)$ with $(s_1', s_2') \in R$.
- If $s_2' \in Post(s_2)$, then there exists an $s_1' \in Post(s_1)$ with $(s_1', s_2') \in R$.

States s_1 and s_2 are bisimulation-equivalent (or bisimilar), denoted $s_1 \sim_{TS} s_2$, if there exists a bisimulation R for TS with $(s_1, s_2) \in R$. The relations $\sim_n \subseteq S \times S$ are inductively defined by:

(a) $s_1 \sim_0 s_2$ iff $L(s_1) = L(s_2)$.

(b) $s_1 \sim_{n+1} s_2$ iff

- $L(s_1) = L(s_2)$,
- for all $s_1' \in Post(s_1)$ there exists $s_2' \in Post(s_2)$ with $s_1' \sim_n s_2'$,
- for all $s_2' \in Post(s2)$ there exists $s_1' \in Post(s_1)$ with $s_1' \sim_n s_2'$.

Show that for finite TS it holds that $\sim_{TS} = \bigcap_{n \geq 0} \sim_n$, i.e., $s_1 \sim_{TS} s_2$ if and only if $s_1 \sim_n s_2$ for all $n \geq 0$.