Technische Universität München (I7) **Summer Semester 2020**
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Model Checking – Exercise sheet 9

Exercise 9.1
Let $a = a_2a_1a_0$, $b = b_2b_1b_0$, and $c = c_3c_2c_1c_0$ be 3-bit, 3-bit, and 4-bit unsigned integers, respectively.

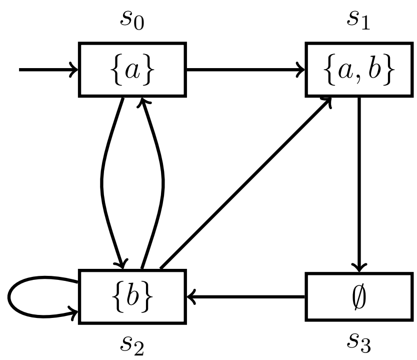
(a) Draw a BDD that represents $a + b = c$. Write down your variable ordering.

(b) Draw a BDD that represents $a = 2 \cdot b$. The BDD should contain every possible value of b such that $2 \cdot b$ is representable using 3 bits. The variable ordering of a and b must be the same as in (a).

(c) Use the BDDs from (a) and (b) to construct a BDD that represents $3 \cdot b = c$.

(d) Use the BDD from (c) to construct a BDD that represents $c \bmod 3 = 0$.

Exercise 9.2



For the given transition system,

(a) Construct a BDD representing the transition system.

(b) Using the BDD from (a), construct the BDD representing

(i) $Img(b)$ where $Img(\phi)$ is the set of successors of states which satisfy the formula ϕ.

(ii) $Pre(a)$ where $Pre(\phi)$ is the set of predecessors of states which satisfy ϕ.

Exercise 9.3
For a given transition system as a BDD T and a set of states as a BDD S, give an algorithm to compute the set of all reachable states from S. Also, Give an algorithm to compute the shortest path between two given states s_1 and s_2 using T.

Solution 9.1

(a) $w_1 = a_0 \wedge b_0$
$w_2 = a_1 \wedge b_1 \vee ((a_1 \oplus b_1) \wedge w_1)$
$w_3 = a_2 \wedge b_2 \vee ((a_2 \oplus b_2) \wedge w_2)$
$adder = (c_0 \iff a_0 \oplus b_0) \wedge (c_1 \iff a_1 \oplus b_1 \oplus w_1) \wedge (c_2 \iff a_2 \oplus b_2 \oplus w_2) \wedge (c_3 \iff w_3)$

(b) $mult = a'_0 \wedge (b_0 \iff a_1) \wedge (b_1 \iff a_2) \wedge b'_2$

(c) $threeb = \exists a_0, a_1, a_2.\ adder \wedge mult$

(d) $mod = \exists b_0, b_1, b_2.\ threeb$

Solution 9.2

(a) $TS = (a \wedge \neg b \wedge b') \vee (a \wedge b \wedge \neg a' \wedge \neg b') \vee (\neg a \wedge b \wedge (a' \wedge b')) \vee (\neg a \wedge \neg b \wedge \neg a' \wedge b')$

(b) (i) $\exists a, b.\ (b \wedge TS)$
(ii) $\exists a', b'.\ (a' \wedge TS)$

Solution 9.3

Reachable states: Start with $S_0 = S$ and inductively compute $S_{i+1} = Img(S_i) \cup S_i$ until $S_{i+1} = S_i$.

Shortest path: Start with $S_0 = s_1$ and inductively compute $S_{i+1} = Img(S_i)$ until $S_{i+1} \wedge s_2 \neq \bot$. Let's assume that at kth step, it stops i.e. $S_k \wedge s_2 \neq \bot$. Now, compute $P_0 = s_2$ and $P_{j+1} = Pre(P_j) \wedge S_{k-j}$ until P_k. Note that, $P_k \wedge s_1 \neq \bot$ and you can find the path from s_1 to s_2 as follows: Pick s_1 from P_k and pick s from P_j such that s is a successor of the element picked from P_{j+1}.