## Model Checking - Exercise sheet 9

## Exercise 9.1

Let $a=a_{2} a_{1} a_{0}, b=b_{2} b_{1} b_{0}$, and $c=c_{3} c_{2} c_{1} c_{0}$ be 3 -bit, 3 -bit, and 4 -bit unsigned integers, respectively.
(a) Draw a BDD that represents $a+b=c$. Write down your variable ordering.
(b) Draw a BDD that represents $a=2 \cdot b$. The BDD should contain every possible value of $b$ such that $2 \cdot b$ is representable using 3 bits. The variable ordering of $a$ and $b$ must be the same as in (a).
(c) Use the BDDs from (a) and (b) to construct a $\operatorname{BDD}$ that represents $3 \cdot b=c$.
(d) Use the BDD from (c) to construct a BDD that represents $c \bmod 3=0$.

## Exercise 9.2

For the given transition system,

(a) Construct a BDD representing the transition system.
(b) Using the BDD from (a), construct the BDD representing
(i) $\operatorname{Img}(b)$ where $\operatorname{Img}(\phi)$ is the set of successors of states which satisfy the formula $\phi$.
(ii) $\operatorname{Pre}(a)$ where $\operatorname{Pre}(\phi)$ is the set of predecessors of states which satisfy $\phi$.

## Exercise 9.3

For a given transition system as a $\mathrm{BDD} T$ and a set of states as a $\mathrm{BDD} S$, give an algorithm to compute the set of all reachable states from $S$. Also, Give an algorithm to compute the shortest path between two given states $s_{1}$ and $s_{2}$ using $T$.

