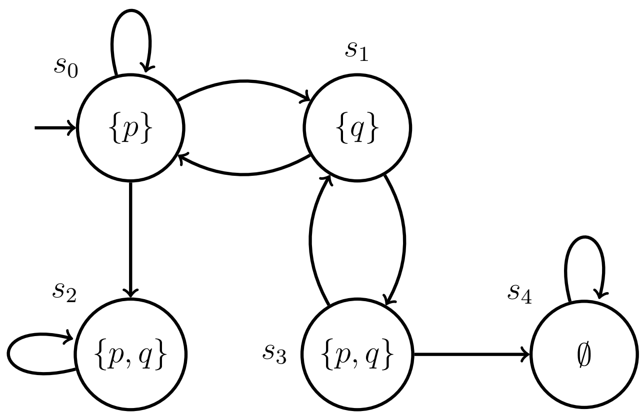
Model Checking – Exercise sheet 7

Exercise 7.1
Compute $[\![\mathbf{EG}q]\!]$, $[\![\mathbf{EXAG}(p \vee q)]\!]$ and $[\![\mathbf{EFAG}(p \wedge q)]\!]$ for the following Kripke structure:



Exercise 7.2
(Taken from *Principles of Model Checking*)
Provide two Kripke structures $\mathcal{K}_1$ and $\mathcal{K}_2$ (over the same set of atomic propositions) and a CTL formula ϕ such that $Traces(\mathcal{K}_1) = Traces(\mathcal{K}_2)$ and $\mathcal{K}_1 \models \phi$, but $\mathcal{K}_2 \not\models \phi$.

Exercise 7.3
Given two CTL formulas ϕ_1 and ϕ_2, we write $\phi_1 \Rightarrow \phi_2$ iff for every Kripke structure $\mathcal{K}$ we have $(\mathcal{K} \models \phi_1) \Rightarrow (\mathcal{K} \models \phi_2)$. Furthermore, we define an *implication graph* as a directed graph whose nodes are CTL formulas, and that contains an edge from ϕ_1 to ϕ_2 iff $\phi_1 \Rightarrow \phi_2$. Let $AP = \{p\}$.

(a) Draw an implication graph with the nodes: $\mathbf{EFEF}p$, $\mathbf{EGEG}p$, $\mathbf{AFAF}p$, $\mathbf{AGAG}p$.

(b) For each implication $\phi_1 \Rightarrow \phi_2$ obtained in (a), give a Kripke structure $\mathcal{K}$ that satisfies ϕ_2 but not ϕ_1, i.e. give a $\mathcal{K}$ such that $\mathcal{K} \models \phi_2$ and $\mathcal{K} \not\models \phi_1$.

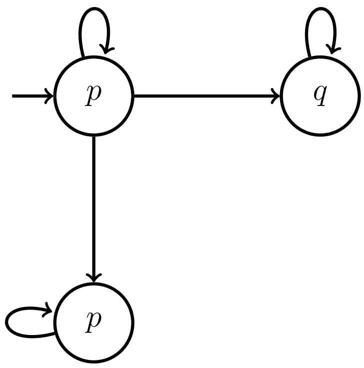
(c) Add the following CTL formulas to the implication graph obtained in (a): $\mathbf{AFEF}p$, $\mathbf{EFAF}p$, $\mathbf{AGEG}p$, $\mathbf{EGAG}p$.

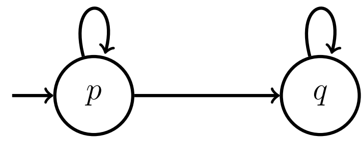
Solution 7.1

1. $[\![\mathbf{EG}q]\!] = \{s_1, s_2, s_3\}$
2. $[\![\mathbf{EXAG}(p \vee q)]\!] = \{s_0\}$
3. $[\![\mathbf{EFAG}(p \wedge q)]\!] = \{s_0, s_1, s_2, s_3\}$

Solution 7.2

Consider the following Kripke structures and the property $\phi = \mathbf{EFAG}p$, first one satisfies ϕ and second one does not.





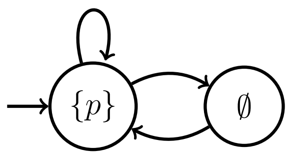
Solution 7.3

Note that the "⇒" relation is transitive, hence all transitive edges in (a), (b) and (d) are omitted.

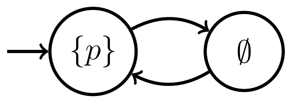
(a)

$$
\begin{tikzcd}
 & & \mathbf{AG}p \equiv \mathbf{AGAG}p \arrow[dll] \\
\mathbf{EG}p \equiv \mathbf{EGEG}p \arrow[dr] & & \\
 & \mathbf{AF}p \equiv \mathbf{AFAF}p \arrow[dr] & \\
 & & \mathbf{EF}p \equiv \mathbf{EFEF}p
\end{tikzcd}
$$

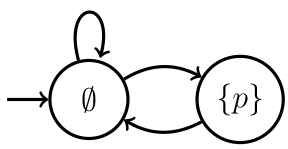
(b) The following Kripke structure satisfies $\mathbf{EG}p$, but not $\mathbf{AG}p$:



The following Kripke structure satisfies $\mathbf{AF}p$, but not $\mathbf{EG}p$:



The following Kripke structure satisfies $\mathbf{EF}p$, but not $\mathbf{AF}p$:



(c)

$$
\begin{tikzcd}
 & \mathbf{AG}p \equiv \mathbf{AGAG}p \equiv \mathbf{AGEG}p \equiv \mathbf{EGAG}p \arrow[dl] \\
\mathbf{EG}p \equiv \mathbf{EGEG}p \arrow[d] & \\
\mathbf{AF}p \equiv \mathbf{AFAF}p \arrow[dr] & \\
 & \mathbf{EF}p \equiv \mathbf{EFEF}p \equiv \mathbf{AFEF}p \equiv \mathbf{EFAF}p
\end{tikzcd}
$$