Technische Universität München (I7) **Summer Semester 2020**
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Model Checking – Exercise sheet 4

Exercise 4.1
Using the *Compare* feature in Spot (`https://spot.lrde.epita.fr/app`) give an LTL formula equivalent to

(a) $p\ \mathbf{R}\ q$, which does not contain $\neg$ but may contain $\mathbf{U}, \mathbf{G}$ or $\mathbf{F}$.

(b) $(\mathbf{G}p)\ \mathbf{U}\ q$ which does not contain $\mathbf{U}$.

(c) $(\mathbf{F}p)\ \mathbf{U}\ q$, which does not contain $\mathbf{U}$.

Exercise 4.2
Think of a way to use Spot to check if a word α satisfies an LTL formula ϕ. Check if the word $\{q\}\emptyset\{s\}\emptyset\{p\}^\omega$ satisfies $\mathbf{G}\neg q \vee \mathbf{F}(q \wedge (\neg p\ \mathbf{W}\ s))$.

Exercise 4.3
Challenge: what is the largest LTL formula you can come up with using only one atomic proposition p and without using the $\mathbf{X}$ operator, which Spot is unable to simplify?

Exercise 4.4
Given the following Kripke structures and LTL formulae, answer the following questions

(a) Which of $\mathcal{K}_1, \mathcal{K}_2$ and $\mathcal{K}_3$ satisfy $\phi = \mathbf{G}(\mathbf{X}q \to p)$?

(b) Give an LTL formula which exactly characterizes $\mathcal{K}_3$, i.e. both the formula and the Kripke structure accept exactly the same words.

Exercise 4.5
Let $AP = \{p, q\}$ and let $\Sigma = 2^{AP}$. Give Büchi automata recognizing the ω-languages over Σ defined by the following LTL formulas:

(a) $\mathbf{XG}\neg p$

(b) $(\mathbf{GF}p) \to (\mathbf{F}q)$

(c) $p \wedge \neg(\mathbf{XF}p)$

(d) $\mathbf{G}(p\ \mathbf{U}\ (p \to q))$

(e) $\mathbf{F}q \to (\neg q\ \mathbf{U}\ (\neg q \wedge p))$

Exercise 4.6
Given $L = \{\{p\}^m\{q\}^n\emptyset^\omega : m \leq n\}$, show that there is no Büchi automata recognizing L.

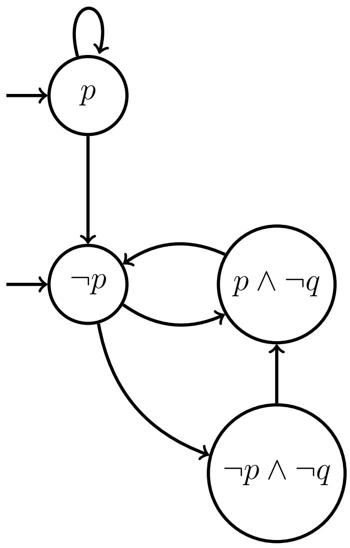


Figure 1: $\mathcal{K}_1$

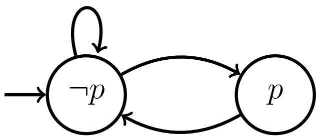


Figure 2: $\mathcal{K}_2$

Solution 4.1

(a) $(q\ \mathbf{U}\ (p \wedge q)) \vee \mathbf{G}q$

(b) $(\mathbf{F}q \wedge \mathbf{G}p) \vee q$.

(c) $q \vee \mathbf{F}(\mathbf{F}p \wedge \mathbf{X}q)$ or
another longer solution: $\mathbf{F}(q \wedge \mathbf{F}p) \vee \mathbf{F}(p \wedge \mathbf{X}q) \vee \mathbf{G}q \vee q$

Solution 4.2

Use $\mathbf{X}$, $\mathbf{XX}$ and so on to describe the word. Then run compare.

Solution 4.3

Solution by Max and Lukas from the tutorial: $(((p\wedge\mathbf{F}\neg p\wedge\mathbf{G}(\mathbf{F}p\wedge\mathbf{F}(\mathbf{G}\neg p|\mathbf{G}p)))|(\mathbf{F}(\mathbf{G}\neg p|\mathbf{G}(\mathbf{F}p\wedge \mathbf{F}\neg p))\wedge(\neg p|\mathbf{G}p)))\wedge((p\wedge\mathbf{F}(\mathbf{G}\neg p|\mathbf{G}(\mathbf{F}p\wedge\mathbf{F}\neg p)))|(p\wedge\mathbf{F}\neg p\wedge(\neg p|\mathbf{G}(\mathbf{F}p\wedge\mathbf{F}(\mathbf{G}\neg p|\mathbf{G}p))))))|((\neg p|\mathbf{G}(\mathbf{F}p\wedge \mathbf{F}(\mathbf{G}\neg p|\mathbf{G}p))) \wedge ((p \wedge \mathbf{F}(\mathbf{G}\neg p|\mathbf{G}(\mathbf{F}p \wedge \mathbf{F}\neg p)))|(\mathbf{G}(\mathbf{F}p \wedge \mathbf{F}(\mathbf{G}\neg p|\mathbf{G}p)) \wedge (\neg p|\mathbf{G}p))))$

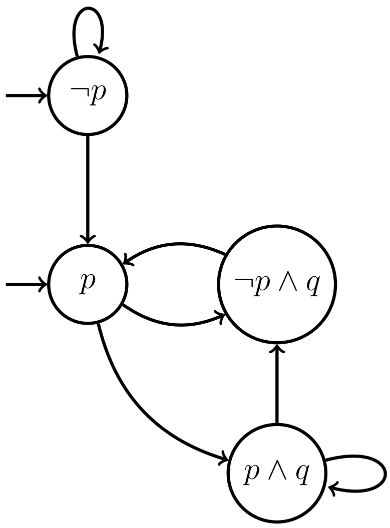


Figure 3: $\mathcal{K}_3$

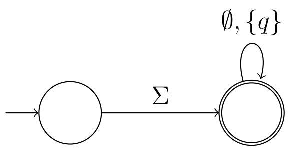
Solution 4.4

(a) $\mathcal{K}_1$

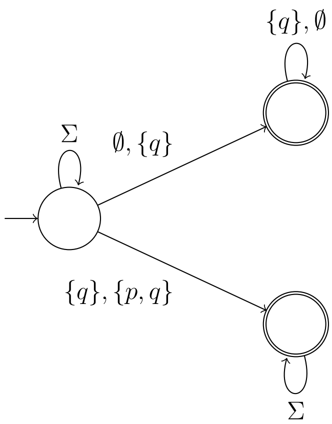
(b) $\mathbf{G}(p \to \mathbf{X}q)$

Solution 4.5

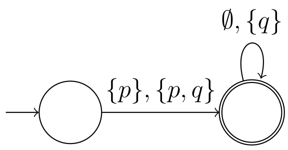
(a)



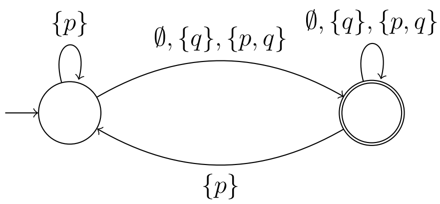
(b) Note that $(\mathbf{GF}p) \to (\mathbf{F}q) \equiv \neg(\mathbf{GF}p) \vee (\mathbf{F}q) \equiv (\mathbf{FG}\neg p) \vee (\mathbf{F}q)$. We construct Büchi automata for $\mathbf{FG}\neg p$ and $\mathbf{F}q$, and take their union:



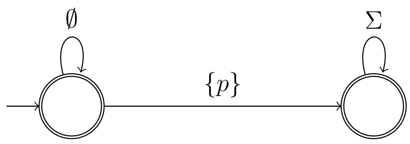
(c) Note that $p \wedge \neg(\mathbf{XF}p) \equiv p \wedge \mathbf{XG}\neg p$. We construct a Büchi automaton for $p \wedge \mathbf{XG}\neg p$:



(d)



(e)



Solution 4.6

For the sake of contradiction, suppose there exists a Büchi automaton $B = (Q, \Sigma, \delta, Q_0, F)$ such that $\mathcal{L}(B) = L$. Let $m = |Q|$ and let $\sigma = \{p\}^m\{q\}^m\emptyset^\omega$. Since $\sigma \in \mathcal{L}(B)$, there exist $q_0, q_1, \ldots \in Q$ such that $q_0 \in Q_0$, there are infinitely many indices i such that $q_i \in F$ and

$$q_0 \xrightarrow{\sigma_0} q_1 \xrightarrow{\sigma_1} q_2 \cdots .$$

By the pigeonhole principle, there exist $0 \leq i < j \leq m$ such that $q_i = q_j$. Let $u = \sigma_0\sigma_1 \cdots \sigma_{i-1}$, $v = \sigma_i\sigma_{i+1} \cdots \sigma_{j-1}$ and $w = \sigma_j\sigma_{j+1} \cdots$. We have:

$$q_0 \xrightarrow{u} q_i \xrightarrow{v^{m+1}} q_j \xrightarrow{w} \cdots$$

Thus, $\sigma' \in \mathcal{L}(B)$ where $\sigma' = uv^{m+1}w$. Note that v solely consists of the letter $\{p\}$, hence $|P_{\sigma'}| \geq m + 1 > m = |Q_{\sigma'}|$, which contradicts $\sigma \in \mathcal{L}(B) = L$.