

Model Checking – Exercise sheet 4

Exercise 4.1

Using the *Compare* feature in Spot (<https://spot.lrde.epita.fr/app>) give an LTL formula equivalent to

- (a) $p \mathbf{R} q$, which does not contain \neg but may contain \mathbf{U} , \mathbf{G} or \mathbf{F} .
- (b) $(\mathbf{G}p) \mathbf{U} q$ which does not contain \mathbf{U} .
- (c) $(\mathbf{F}p) \mathbf{U} q$, which does not contain \mathbf{U} .

Exercise 4.2

Think of a way to use Spot to check if a word α satisfies an LTL formula ϕ . Check if the word $\{q\}\emptyset\{s\}\emptyset\{p\}^\omega$ satisfies $\mathbf{G}\neg q \vee \mathbf{F}(q \wedge (\neg p \mathbf{W} s))$.

Exercise 4.3

Challenge: what is the largest LTL formula you can come up with using only one atomic proposition p and without using the \mathbf{X} operator, which Spot is unable to simplify?

Exercise 4.4

Given the following Kripke structures and LTL formulae, answer the following questions

- (a) Which of $\mathcal{K}_1, \mathcal{K}_2$ and \mathcal{K}_3 satisfy $\phi = \mathbf{G}(\mathbf{X}q \rightarrow p)$?
- (b) Give an LTL formula which exactly characterizes \mathcal{K}_3 , i.e. both the formula and the Kripke structure accept exactly the same words.

Exercise 4.5

Let $AP = \{p, q\}$ and let $\Sigma = 2^{AP}$. Give Büchi automata recognizing the ω -languages over Σ defined by the following LTL formulas:

- (a) $\mathbf{XG}\neg p$
- (b) $(\mathbf{GF}p) \rightarrow (\mathbf{F}q)$
- (c) $p \wedge \neg(\mathbf{XF}p)$
- (d) $\mathbf{G}(p \mathbf{U} (p \rightarrow q))$
- (e) $\mathbf{F}q \rightarrow (\neg q \mathbf{U} (\neg q \wedge p))$

Exercise 4.6

Given $L = \{\{p\}^m \{q\}^n \emptyset^\omega : m \leq n\}$, show that there is no Büchi automata recognizing L .

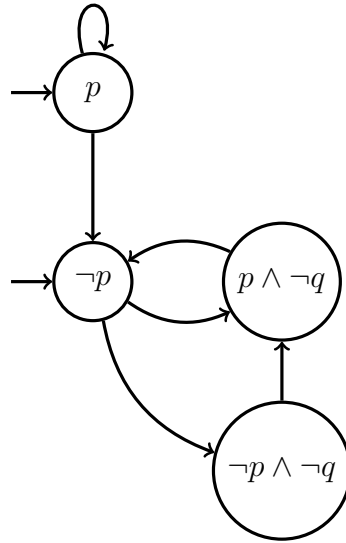


Figure 1: \mathcal{K}_1

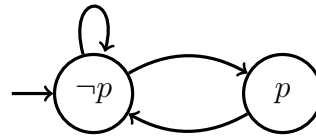


Figure 2: \mathcal{K}_2

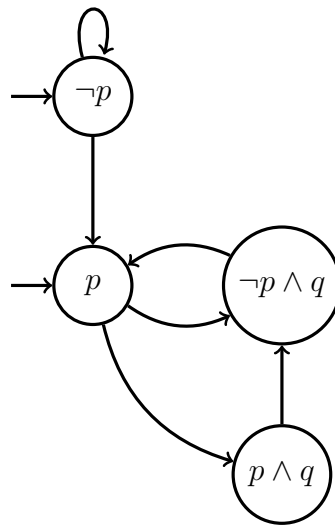


Figure 3: \mathcal{K}_3