

## Model Checking – Exercise sheet 7

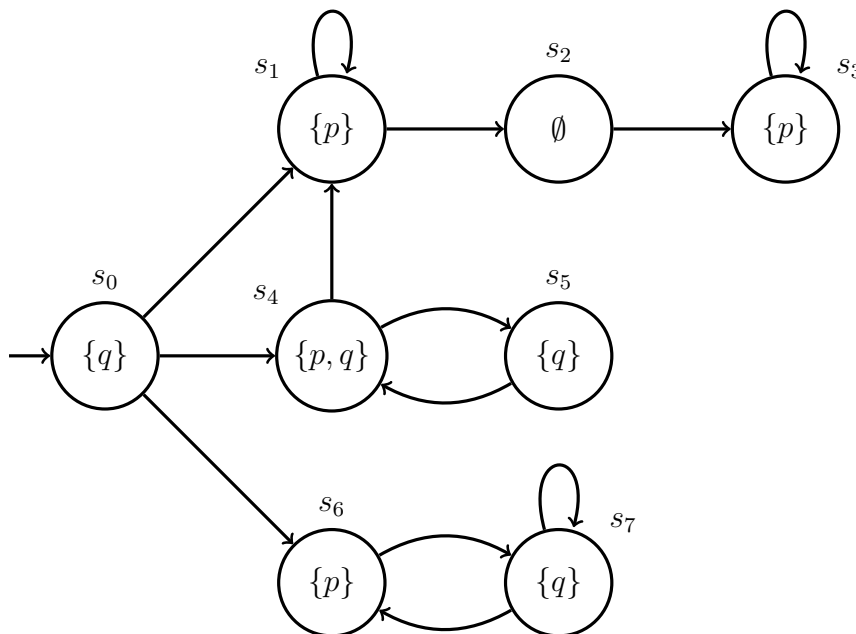
### Exercise 7.1

Given two CTL formulas  $\phi_1$  and  $\phi_2$ , we write  $\phi_1 \Rightarrow \phi_2$  iff for every Kripke structure  $\mathcal{K}$  we have  $(\mathcal{K} \models \phi_1) \Rightarrow (\mathcal{K} \models \phi_2)$ . Furthermore, we define an *implication graph* as a directed graph whose nodes are CTL formulas, and that contains an edge from  $\phi_1$  to  $\phi_2$  iff  $\phi_1 \Rightarrow \phi_2$ . Let  $AP = \{p\}$ .

- (a) Draw an implication graph with the nodes: **EFEF** $p$ , **EGEG** $p$ , **AFAF** $p$ , **AGAG** $p$ .
- (b) For each implication  $\phi_1 \Rightarrow \phi_2$  obtained in (a), give a Kripke structure  $\mathcal{K}$  that satisfies  $\phi_2$  but not  $\phi_1$ , i.e. give a  $\mathcal{K}$  such that  $\mathcal{K} \models \phi_2$  and  $\mathcal{K} \not\models \phi_1$ .
- (c) Add the following CTL formulas to the implication graph obtained in (a): **AFEF** $p$ , **EFAF** $p$ , **AGEG** $p$ , **EGAG** $p$ .
- (d) Complete the graph obtained in (c) with the nodes: **AGAF** $p$ , **AFAG** $p$ , **AGEF** $p$ , **EGAF** $p$ , **AFEG** $p$ , **EFAG** $p$ , **EFEG** $p$ , **EGEF** $p$ .

### Exercise 7.2

Consider the following Kripke structure over  $AP = \{p, q\}$ :



- (a) Compute  $\llbracket \mathbf{EG}q \rrbracket$  and  $\llbracket \mathbf{EF}q \rrbracket$ .
- (b) Compute  $\llbracket \mathbf{AGAF}p \rrbracket$  and  $\llbracket \mathbf{EFAG}\neg q \rrbracket$ .