

Model Checking – Exercise sheet 5

Exercise 5.1

Extend the set of rules of the LTL to Büchi automata translation to directly deal with the **F** and **G** operators. Use it to construct a Büchi automaton for $\phi = \mathbf{GF}p$. Is it necessary to construct states which do not contain ϕ ?

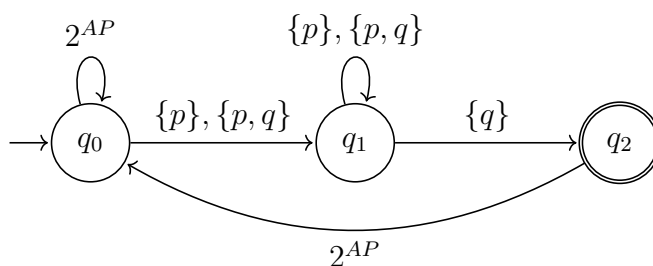
Exercise 5.2

Let $\phi = \mathbf{G}((\mathbf{X}(p \mathbf{U} q)) \rightarrow ((\neg p \wedge \mathbf{F}q) \vee (q \mathbf{U} \mathbf{X}q)))$ and \mathcal{G} be a generalized Büchi automaton translated from ϕ using the construction presented in the lecture and the extended set of rules defined in the previous exercise.

- (a) Write down the set of subformulae $Sub(\phi)$.
- (b) What is the size of $CS(\phi)$?
- (c) How many sets of accepting states does \mathcal{G} have?
- (d) Is $\{\phi\}$ an accepting state of \mathcal{G} ?
- (e) Give a reachable state that has no successors.
- (f) Give a successor state of the smallest consistent state containing $\{\phi, q, q \mathbf{U} \mathbf{X}q, \mathbf{F}q\}$.
- (g) Give a predecessor state of the smallest consistent state containing $\{\phi, q, q \mathbf{U} \mathbf{X}q, \mathbf{F}q\}$.

Exercise 5.3

Consider the following Büchi automaton \mathcal{B} :



- (a) Give an LTL formula ϕ such that $\mathcal{L}(\mathcal{B}) = \llbracket \phi \rrbracket$.
- (b) By using the result from (a), propose a method to construct a Büchi automaton that accepts the complement of the language accepted by \mathcal{B} .

- (c) Construct a Büchi automaton for the formula $\mathbf{G}(\neg p \vee (\neg p \mathbf{R} (p \vee \neg q)))$.
- (d) By using the result from (c), construct a Büchi automaton that accepts the complement of the language accepted by \mathcal{B} .

Solution 5.1

Extend the definition of NNF to include **F** and **G**, extend the corresponding $Sub(\phi)$:

- if $\mathbf{F}\phi_1 \in Sub(\phi)$ then $\phi_1 \in Sub(\phi)$
- if $\mathbf{G}\phi_1 \in Sub(\phi)$ then $\phi_1 \in Sub(\phi)$,

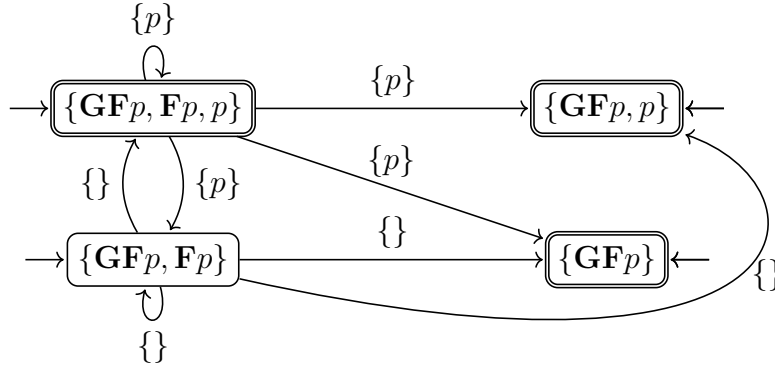
and extend rules for transitions as follows: $(M, \sigma, M') \in \Delta$ iff $\sigma = M \cap AP$ and

- if $\mathbf{F}\phi_1 \in Sub(\phi)$, then $\mathbf{F}\phi_1 \in M$ iff $\phi_1 \in M$ or $\mathbf{F}\phi_1 \in M'$
- if $\mathbf{G}\phi_1 \in Sub(\phi)$, then $\mathbf{G}\phi_1 \in M$ iff $\phi_1 \in M$ and $\mathbf{G}\phi_1 \in M'$

Also, the acceptance condition must be extended for **F**: \mathcal{F} contains a set F_ψ , for every subformula ψ of the form $\mathbf{F}\phi_1$, where

$$F_\psi = \{M \in CS(\phi) \mid \phi_1 \in M \text{ or } \neg(\mathbf{F}\phi_1) \in M\}$$

The translated Büchi automaton for $\phi = \mathbf{GF}p$ is below. Notice that the initial states must contain $\mathbf{GF}p$, and from the translation rule successors of states with $\mathbf{GF}p$ must also contain $\mathbf{GF}p$. So, it is not necessary to construct states without $\mathbf{GF}p$.



Solution 5.2

Translate ϕ into an NNF formula:

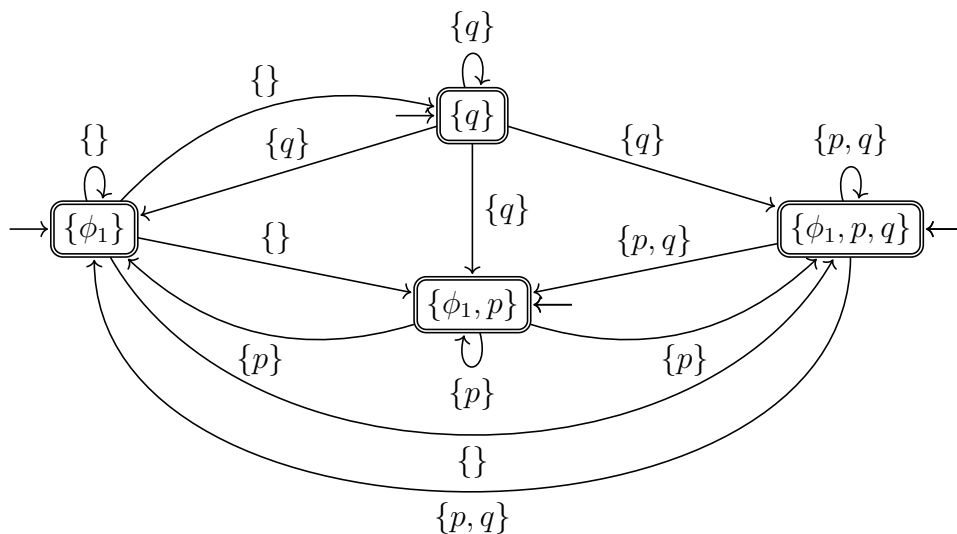
$$\begin{aligned} \phi &= \mathbf{G}((\mathbf{X}(p \mathbf{U} q)) \rightarrow ((\neg p \wedge \mathbf{F}q) \vee (q \mathbf{U} \mathbf{X}q))) \\ &\equiv \mathbf{G}((\mathbf{X}(\neg p \mathbf{R} \neg q)) \vee ((\neg p \wedge \mathbf{F}q) \vee (q \mathbf{U} \mathbf{X}q))) \end{aligned}$$

- (a) Let $\phi_1 = \neg p \mathbf{R} \neg q$, $\phi_2 = \neg p \wedge \mathbf{F}q$, and $\phi_3 = q \mathbf{U} \mathbf{X}q$. We have $\phi = \mathbf{G}(\mathbf{X}\phi_1 \vee (\phi_2 \vee \phi_3))$ and $Sub(\phi) = \{\mathbf{true}, \phi, \mathbf{X}\phi_1 \vee (\phi_2 \vee \phi_3), \mathbf{X}\phi_1, \phi_2 \vee \phi_3, \phi_1, \phi_2, \phi_3, \mathbf{F}q, \mathbf{X}q, p, q\} \cup \{\mathbf{false}, \neg\phi, \neg(\mathbf{X}\phi_1 \vee (\phi_2 \vee \phi_3)), \neg\mathbf{X}\phi_1, \neg(\phi_2 \vee \phi_3), \neg\phi_1, \neg\phi_2, \neg\phi_3, \neg\mathbf{F}q, \neg\mathbf{X}q, \neg p, \neg q\}$.
- (b) Only $\phi, \mathbf{X}\phi_1, \phi_1, \phi_3, \mathbf{F}q, \mathbf{X}q, p, q$ can independently form consistent states. So, $|CS(\phi)| = 2^8 = 256$ states

- (c) $\mathcal{F} = \{F_{q \mathbf{U} \mathbf{X} q}, F_{\mathbf{F} q}\}$
- (d) $\{\phi\} \in F_{q \mathbf{U} \mathbf{X} q}$ and $\{\phi\} \in F_{\mathbf{F} q}$
- (e) $\{\phi\}$ is reachable because it is an initial state, and it has no successors because $\mathbf{X}\phi_1 \vee (\phi_2 \vee \phi_3) \notin \{\phi\}$.
- (f) $\{\phi, q \mathbf{U} \mathbf{X} q\}$
- (g) $\{\phi, q, q \mathbf{U} \mathbf{X} q, \mathbf{F} q, \mathbf{X} q\}$

Solution 5.3

- (a) $\phi = \mathbf{GF}(p \wedge (p \mathbf{U} (\neg p \wedge q)))$
- (b) Construct a Büchi automaton for $\neg\phi$ by using e.g. the translation in the lecture.
- (c) Let $\phi_1 = \neg p \mathbf{R} (p \vee \neg q)$. Note that ϕ_1, p, q are enough to form consistent sets, i.e. we assume that ϕ and $\neg p \vee (\neg p \mathbf{R} (p \vee \neg q))$ are implicitly in every state. So, $CS(\phi) = 2^{\{\phi_1, p, q\}}$. However, we know that $\{p\}$ and $\{p, q\}$ have no successors because of \mathbf{G} , and $\{\}$ and $\{\phi_1, q\}$ have no successors because of \mathbf{R} .



- (d) Notice that $\neg\phi \equiv \mathbf{FG}(\neg p \vee (\neg p \mathbf{R} (p \vee \neg q)))$. It suffices to add a self-looping initial state and transitions from it to all states in (c).