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Model Checking – Exercise sheet 4

Exercise 4.1

Let $AP = \{p, q\}$ and let $\Sigma = 2^{AP}$. Give Büchi automata recognizing the ω-languages over Σ defined by the following LTL formulas:

(a) $\mathbf{XG}\neg p$

(b) $(\mathbf{GF}p) \rightarrow (\mathbf{F}q)$

(c) $p \wedge \neg(\mathbf{XF}p)$

(d) $\mathbf{G}(p \ \mathbf{U} \ (p \rightarrow q))$

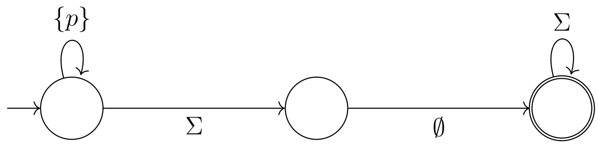
(e) $\mathbf{F}q \rightarrow (\neg q \ \mathbf{U} \ (\neg q \wedge p))$

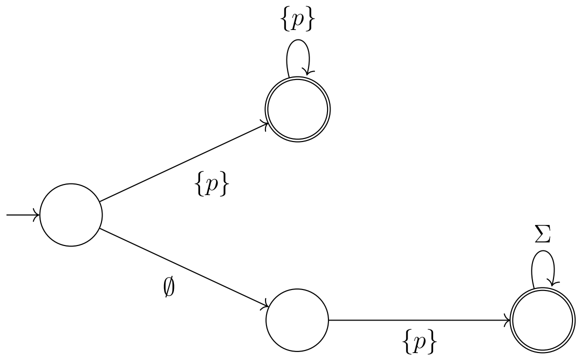
Exercise 4.2

Given $L = \{\{p\}^m\{q\}^n\emptyset^\omega : m \leq n\}$, show that there is no Büchi automata recognizing L. [Hint:]

Exercise 4.3

Let $AP = \{p\}$. Given two Büchi automata recognizing ω-regular languages over $\Sigma = 2^{AP}$, prove or disprove that one is the negation of the other. [Hint:]

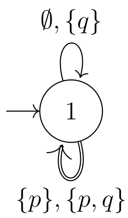




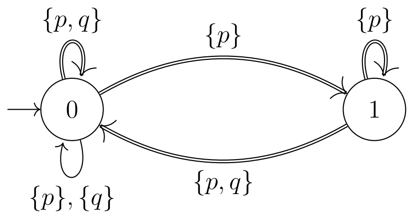
Exercise 4.4
Given $AP = \{p\}$, come up with an LTL formula (without $\mathbf{X}$) over $\Sigma = 2^{AP}$ which might have the largest automaton (use Spot).

Exercise 4.5
Convert the following Büchi automata with transition-based acceptance condition ("doubled"-transitions have to be seen infinitely often) to equivalent Büchi automata with state-based acceptance conditions. Moreover, give a general procedure to perform this conversion.

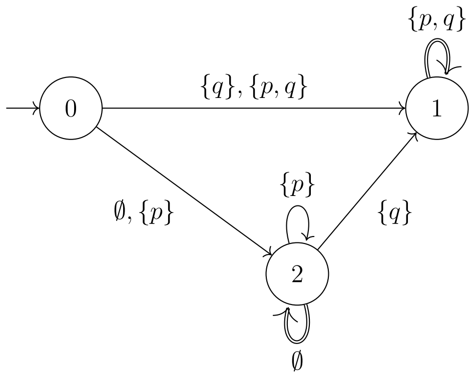
(a)



(b)

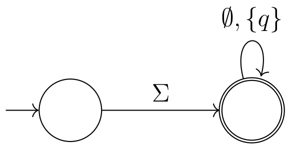


(c)

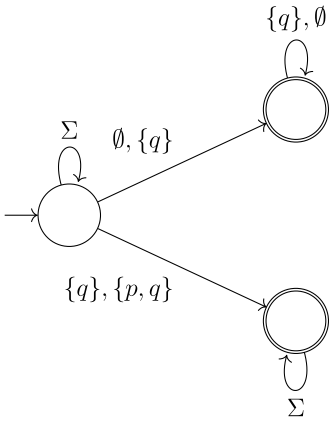


Solution 4.1

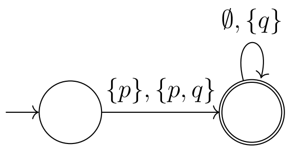
(a)



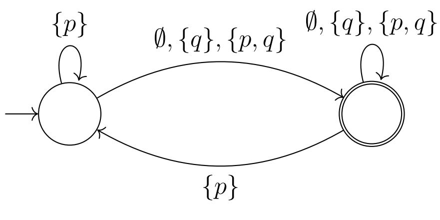
(b) Note that $(\mathbf{GF}p) \rightarrow (\mathbf{F}q) \equiv \neg(\mathbf{GF}p) \vee (\mathbf{F}q) \equiv (\mathbf{FG}\neg p) \vee (\mathbf{F}q)$. We construct Büchi automata for $\mathbf{FG}\neg p$ and $\mathbf{F}q$, and take their union:



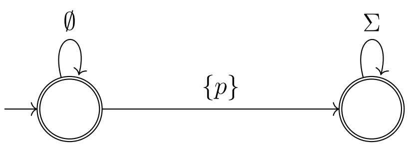
(c) Note that $p \wedge \neg(\mathbf{XF}p) \equiv p \wedge \mathbf{XG}\neg p$. We construct a Büchi automaton for $p \wedge \mathbf{XG}\neg p$:



(d)



(e)



Solution 4.2
For the sake of contradiction, suppose there exists a Büchi automaton $B = (Q, \Sigma, \delta, Q_0, F)$ such that $\mathcal{L}(B) = L$. Let $m = |Q|$ and let $\sigma = \{p\}^m\{q\}^m\emptyset^\omega$. Since $\sigma \in \mathcal{L}(B)$, there exist $q_0, q_1, \ldots \in Q$ such that $q_0 \in Q_0$, there are infinitely many indices i such that $q_i \in F$ and

$$q_0 \xrightarrow{\sigma_0} q_1 \xrightarrow{\sigma_1} q_2 \cdots .$$

By the pigeonhole principle, there exist $0 \leq i < j \leq m$ such that $q_i = q_j$. Let $u = \sigma_0\sigma_1\cdots\sigma_{i-1}$, $v = \sigma_i\sigma_{i+1}\cdots\sigma_{j-1}$ and $w = \sigma_j\sigma_{j+1}\cdots$. We have:

$$q_0 \xrightarrow{u} q_i \xrightarrow{v^{m+1}} q_j \xrightarrow{w} \cdots$$

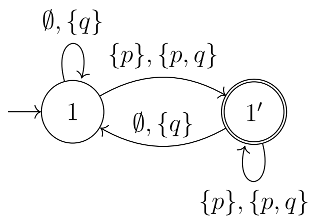
Thus, $\sigma' \in \mathcal{L}(B)$ where $\sigma' = uv^{m+1}w$. Note that v solely consists of the letter $\{p\}$, hence $|P_{\sigma'}| \geq m+1 > m = |Q_{\sigma'}|$, which contradicts $\sigma \in \mathcal{L}(B) = L$.

Solution 4.3
The first Büchi automata accepts the language defined by $p\,\mathbf{U}\,\neg\mathbf{X}p$ and the second accepts the language defined by $(p \wedge \mathbf{XG}p) \vee (\neg p \wedge \mathbf{X}p)$. Using Spot, one may check that one of them is equivalent to the other's complement.

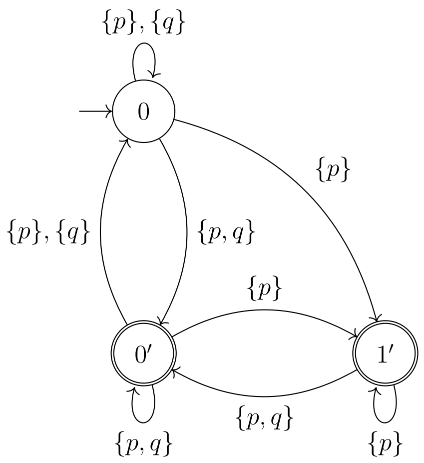
Solution 4.5
The general procedure is as follows.

Let the states of the original automaton be relabeled to $S \times \{1\}$ and create a copy of the states labeled by $S \times \{2\}$. For every accepting transition from $(s_1, 1) \to (s_2, 1)$, change the destination to $(s_2, 2)$. For every non-accepting transition from $(s_1, 2) \to (s_2, 2)$, change the destination to $(s_2, 1)$.

(a)



(b)



(c)