Technische Universität München (I7) **Summer Semester 2019**
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Model Checking – Exercise sheet 3

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Exercise 3.1
Using the *Compare* feature in Spot (`https://spot.lrde.epita.fr/app`) give an LTL formula equivalent to

(a) $p\ \mathbf{R}\ q$, which does not contain $\neg$ but may contain $\mathbf{U}, \mathbf{G}$ or $\mathbf{F}$.

(b) $(\mathbf{G}p)\ \mathbf{U}\ q$ which does not contain $\mathbf{U}$.

(c) $(\mathbf{F}p)\ \mathbf{U}\ q$, which does not contain $\mathbf{U}$.

Exercise 3.2
Discuss the difference between the following LTL formulae in words.

(a) $\mathbf{G}(q \wedge \neg r \rightarrow (\neg r\ \mathbf{W}\ (p \wedge \neg r)))$ and $\mathbf{G}(q \wedge \neg r \rightarrow (\neg r\ \mathbf{U}\ (p \wedge \neg r)))$

(b) $\mathbf{G}((q \wedge \neg r \wedge \mathbf{F}r) \rightarrow (p\ \mathbf{U}\ r))$ and $\mathbf{G}(q \wedge \neg r \rightarrow (p\ \mathbf{W}\ r))$

Exercise 3.3
Think of a way to use Spot to check if a word α satisfies an LTL formula ϕ. Check if the word $\{q\}\emptyset\{s\}\emptyset\{p\}^\omega$ satisfies $\mathbf{G}\neg q \vee \mathbf{F}(q \wedge (\neg p\ \mathbf{W}\ s))$.

Exercise 3.4
Challenge: what is the largest LTL formula you can come up with using only one atomic proposition p and without using the $\mathbf{X}$ operator, which Spot is unable to simplify?

Exercise 3.5
Convert the following ω-regular expressions to LTL formulae. A literal character $x \in \Sigma$ denotes the set containing only x and ϵ denotes the set containing only the empty string. x^* denotes the *Kleene star* operation on x, $x + y$ denotes *alternation* (*or* operator) and . (dot) operator denotes *concatenation*.

(a) $\{p\}^*.\{q\}^*.\{p\}^\omega$

(b) $((\{s\} + \epsilon)^* + \{t\}.(\{t\} + \epsilon)^*.\{s\})^\omega$

(c) $(\{p\}.\{p\}^*.\epsilon^*)^\omega$

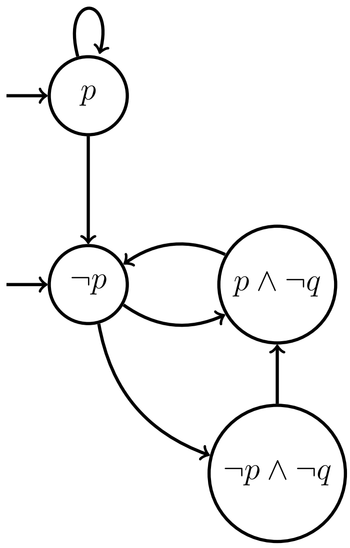

Figure 1: $\mathcal{K}_1$

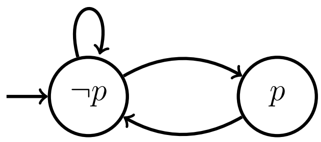

Figure 2: $\mathcal{K}_2$

Exercise 3.6
Given the following Kripke structures and LTL formulae, answer the following questions

(a) Which of $\mathcal{K}_1, \mathcal{K}_2$ and $\mathcal{K}_3$ satisfy $\phi = \mathbf{G}(\mathbf{X}q \rightarrow p)$?

(b) Give an LTL formula which exactly characterizes $\mathcal{K}_3$, i.e. both the formula and the Kripke structure accept exactly the same words.

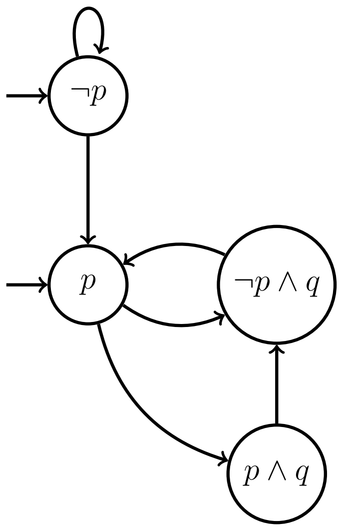

Figure 3: $\mathcal{K}_3$

Solution 3.1
(a) $(q\ \mathbf{U}\ (p \wedge q)) \vee \mathbf{G}q$

(b) $(\mathbf{F}q \wedge \mathbf{G}p) \vee q$.

(c) $\mathbf{F}(q \wedge \mathbf{F}p) \vee \mathbf{F}(p \wedge \mathbf{X}q) \vee \mathbf{G}q \vee q$ or
better solution from class: $q \vee \mathbf{F}(\mathbf{F}p \wedge \mathbf{X}q)$

Solution 3.2
See `http://patterns.projects.cs.ksu.edu/documentation/patterns/ltl.shtml`

(a) (Existence) p becomes true between q and r vs p becomes true after q until r

(b) (Universality) p is true between q and r vs p is true after q until r.

Solution 3.3
Use $\mathbf{X}$, $\mathbf{XX}$ and so on to describe the word. Then run compare.

Solution 3.4
$p\ \mathbf{U}\ (\neg p \wedge (\neg p\ \mathbf{U}\ (p \wedge (p\ \mathbf{U}\ (\neg p \wedge (\neg p\ \mathbf{U}\ p))))))$

Solution 3.5
(a) $p\ \mathbf{U}\ (q\ \mathbf{U}\ \mathbf{G}p)$

(b) $\mathbf{G}(t \rightarrow \mathbf{XF}s)$

(c) $p \wedge \mathbf{XGF}p$

Solution 3.6

(a) $\mathcal{K}_1$

(b) $\mathbf{G}(p \rightarrow \mathbf{X}q)$