Technische Universität München (I7) **Summer Semester 2019**
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Model Checking – Exercise sheet 3

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Exercise 3.1
Using the *Compare* feature in Spot (`https://spot.lrde.epita.fr/app`) give an LTL formula equivalent to

(a) $p \mathbf{R} q$, which does not contain $\neg$ but may contain $\mathbf{U}, \mathbf{G}$ or $\mathbf{F}$.

(b) $(\mathbf{G}p) \mathbf{U} q$ which does not contain $\mathbf{U}$.

(c) $(\mathbf{F}p) \mathbf{U} q$, which does not contain $\mathbf{U}$.

Exercise 3.2
Discuss the difference between the following LTL formulae in words.

(a) $\mathbf{G}(q \land \neg r \to (\neg r \mathbf{W} (p \land \neg r)))$ and $\mathbf{G}(q \land \neg r \to (\neg r \mathbf{U} (p \land \neg r)))$

(b) $\mathbf{G}((q \land \neg r \land \mathbf{F}r) \to (p \mathbf{U} r))$ and $\mathbf{G}(q \land \neg r \to (p \mathbf{W} r))$

Exercise 3.3
Think of a way to use Spot to check if a word α satisfies an LTL formula ϕ. Check if the word $\{q\}\emptyset\{s\}\emptyset\{p\}^\omega$ satisfies $\mathbf{G}\neg q \lor \mathbf{F}(q \land (\neg p \mathbf{W} s))$.

Exercise 3.4
Challenge: what is the largest LTL formula you can come up with using only one atomic proposition p and without using the $\mathbf{X}$ operator, which Spot is unable to simplify?

Exercise 3.5
Convert the following ω-regular expressions to LTL formulae. A literal character $x \in \Sigma$ denotes the set containing only x and ϵ denotes the set containing only the empty string. x^* denotes the *Kleene star* operation on x, $x + y$ denotes *alternation* (*or* operator) and . (dot) operator denotes *concatenation*.

(a) $\{p\}^*.\{q\}^*.\{p\}^\omega$

(b) $((\{s\} + \epsilon)^* + \{t\}.(\{t\} + \epsilon)^*.\{s\})^\omega$

(c) $(\{p\}.\{p\}^*.\epsilon^*)^\omega$

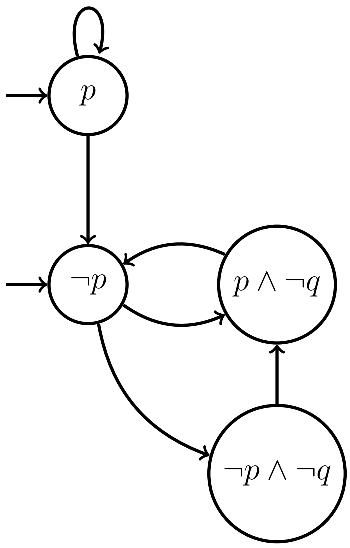

Figure 1: $\mathcal{K}_1$

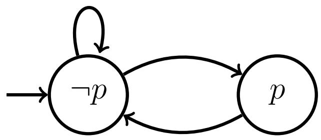

Figure 2: $\mathcal{K}_2$

Exercise 3.6
Given the following Kripke structures and LTL formulae, answer the following questions

(a) Which of $\mathcal{K}_1, \mathcal{K}_2$ and $\mathcal{K}_3$ satisfy $\phi = \mathbf{G}(\mathbf{X}q \to p)$?

(b) Give an LTL formula which exactly characterizes $\mathcal{K}_3$, i.e. both the formula and the Kripke structure accept exactly the same words.

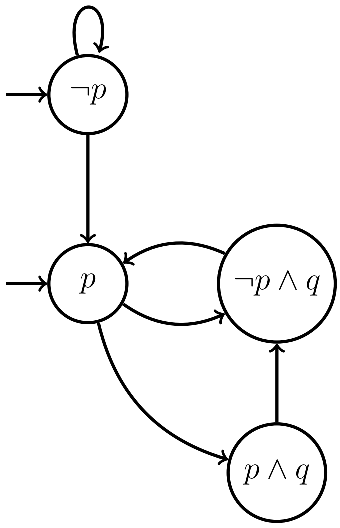

Figure 3: $\mathcal{K}_3$