

Model Checking – Exercise sheet 2

Exercise 2.1

Let $\varphi = \mathbf{GF}p \rightarrow \mathbf{FG}(q \vee r)$ and $\psi = (r \mathbf{U} \mathbf{X}p) \mathbf{U} (q \wedge \neg \mathbf{X} \mathbf{X}s)$ be LTL formulas over the atomic propositions $AP = \{p, q, r, s\}$. Say whether the following sequences satisfy φ and ψ . Justify your answers.

- | | |
|-----------------------------|---|
| (a) \emptyset^ω | (f) $\{r\}\emptyset\{p, q, s\}^\omega$ |
| (b) $\{p, q, r, s\}^\omega$ | (g) $\{r\}\emptyset(\{p, q\}\{r, s\})^\omega$ |
| (c) $\{p\}^\omega$ | (h) $\{r\}\emptyset\{p\}\{q, r\}(\{p, s\}\emptyset)^\omega$ |
| (d) $\{q\}^\omega$ | (i) $\{r\}\emptyset\{p\}\{p, q, r\}(\{s\}\emptyset)^\omega$ |
| (e) $\{p, q\}^\omega$ | (j) $\{q, r\}\emptyset\{p, q\}\emptyset\{r, s\}^\omega$ |

Exercise 2.2

Let $AP = \{s, r, g\}$ be actions of a process: sending a message, receiving a message, and giving a result, respectively. Specify the following properties in LTL, and give example sequences that satisfy and violate the formulas.

- The process always gives a result.
- The process stops communicating after giving its result.
- The process sends infinitely many messages.
- The process only gives a result once.
- The process receives a message after it sends one.
- The process does nothing until it receives a message.

Exercise 2.3

Let $AP = \{p, q\}$. An LTL formula is a tautology if it is satisfied by all sequences over 2^{AP} . Which of the following LTL formulas are tautologies? Justify each answer with a counterexample or a proof.

(a) $\mathbf{G}p \rightarrow \mathbf{F}p$

(d) $\neg\mathbf{F}p \rightarrow \mathbf{F}\neg\mathbf{F}p$

(b) $\mathbf{G}(p \rightarrow q) \rightarrow (\mathbf{G}p \rightarrow \mathbf{G}q)$

(e) $\neg(p \mathbf{U} q) \leftrightarrow (\neg p \mathbf{U} \neg q)$

(c) $\mathbf{F}\mathbf{G}p \vee \mathbf{F}\mathbf{G}\neg p$

(f) $(\mathbf{G}p \rightarrow \mathbf{F}q) \leftrightarrow (p \mathbf{U} (p \vee q))$