Technische Universität München **Summer term 2018**
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Model checking — Endterm

- You have **120 minutes** to complete the exam.
- Answers must be written in a **separate booklet**. Do not answer on the exam.
- Please let us know if you need more paper.
- Write your name and Matrikelnummer on every sheet.
- Write with a non-erasable **pen**. Do not use red or green.
- You are not allowed to use auxiliary means other than pen and paper.
- You can obtain **40 points (plus 4 bonus points)**. You need **17 points** to pass.

Question 1: LTL and Büchi automata $(2+2+2+3=9$ points)

Consider the following LTL formulae over the set of atomic propositions $AP = \{p, q\}$:

$$\phi_1 = \mathbf{FG}(p \;\mathbf{U}\; q) \qquad \phi_2 = \mathbf{FG}(\neg p \to q) \qquad \phi_3 = \mathbf{G}(\neg p \vee (p \;\mathbf{R}\; q))$$

(a) Is there a word satisfying ϕ_1 but not ϕ_2? If so, exhibit such a word and if not, briefly explain why it does not exist.

(b) Is there a word satisfying ϕ_2 but not ϕ_1? If so, exhibit such a word and if not, briefly explain why it does not exist.

(c) Is there a word satisfying all three formulae? If so, exhibit such a word and if not, briefly explain why it does not exist.

(d) Give a Büchi automaton accepting exactly the words satisfying ϕ_1. Make sure it accepts the following words: $\{p, q\}^\omega, \{p\}\{q\}^\omega$ and rejects the following words: $\emptyset^\omega, \{p\}^\omega$.

Question 2: CTL $(1+1+1+1=4$ points)

Consider the CTL formulas $\mathbf{EF}p, \mathbf{EFAG}p, \mathbf{AGEF}p, \mathbf{AGAF}p, \mathbf{AG}p$ over $AP = \{p\}$. Draw:

(a) a Kripke structure $\mathcal{K}_1$ satisfying $\mathbf{EF}p$ but not $\mathbf{EFAG}p$;

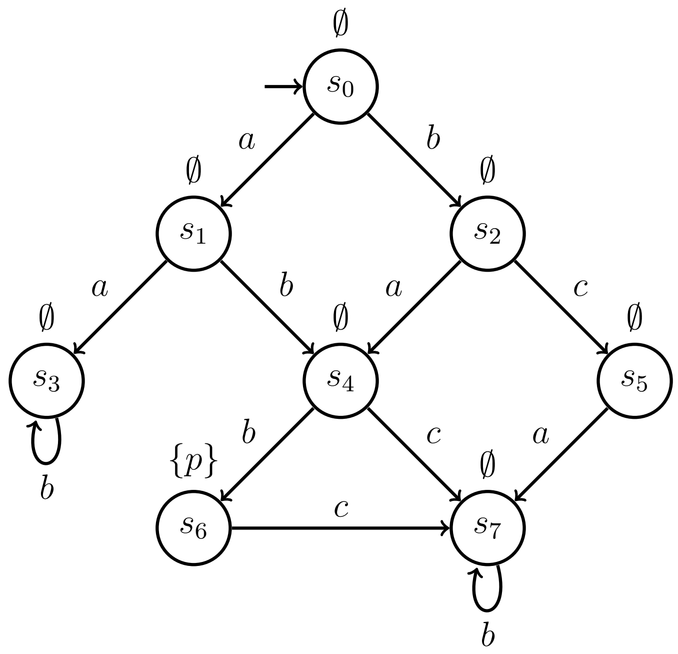
(b) a Kripke structure $\mathcal{K}_2$ satisfying $\mathbf{EFAG}p$ but not $\mathbf{AGEF}p$;

(c) a Kripke structure $\mathcal{K}_3$ satisfying $\mathbf{AGEF}p$ but not $\mathbf{AGAF}p$;

(d) a Kripke structure $\mathcal{K}_4$ satisfying $\mathbf{AGAF}p$ but not $\mathbf{AG}p$.

Question 3: Partial order reduction **($1+1+1+1+1=5$ points)**

Consider the labelled Kripke structure $\mathcal{K} = (S, A, \rightarrow, r, AP, \nu)$ where $S = \{s_0, \ldots, s_7\}$, $A = \{a, b, c\}$ (A is the set of actions), $r = s_0$, $AP = \{p\}$, and $\rightarrow$ and ν are graphically represented below. Observe that p holds only at state s_6 and nowhere else.



(a) Give the largest relation $I \subseteq A \times A$ satisfying the three properties of an independence relation (irreflexivity, symmetry, and the "diamond property") and explain why it is the largest.

(b) Give the largest invisibility set $U \subseteq A$.

(c) Does $\mathit{red}(s_0) = \{a\}$ satisfy condition C_1 (see below) for I and U? Justify your answer.

(d) Does $\mathit{red}(s_4) = \{b\}$ satisfy all of C_0–C_3 (see below) for I and U? Justify your answer.

(e) Does $\mathit{red}(s_2) = \{a\}$ satisfy all of C_0–C_3 (see below) for I and U? Justify your answer.

Recall that the conditions C_0–C_3 for $\mathit{red}(s)$ are:

- C_0: $\mathit{red}(s) = \emptyset$ iff $\mathit{en}(s) = \emptyset$.
- C_1: Every path starting at s satisfies: no action dependent on some action in $\mathit{red}(s)$ can be executed without an action from $\mathit{red}(s)$ occurring first.
- C_2: If $\mathit{red}(s) \neq \mathit{en}(s)$, then all actions in $\mathit{red}(s)$ are invisible.
- C_3: For all cycles in the reduced Kripke structure the following holds: if $a \in \mathit{en}(s)$ for some state s in the cycle, then $a \in \mathit{red}(s')$ for some (possibly other) state s' in the cycle.

Question 4: Binary decision diagrams (4 points)

Assume that you are given a Kripke structure with states $S = \{s_0, s_1, \ldots, s_7\}$.

(a) Compute a multi-BDD representing the two subsets of states $P = \{s_0, s_1, s_3, s_5, s_7\}$ and $Q = \{s_0, s_2, s_6, s_7\}$. Encode each state of S using three bits in the obvious way:

$$s_0 \mapsto 000, s_1 \mapsto 001, \ldots, s_7 \mapsto 111.$$

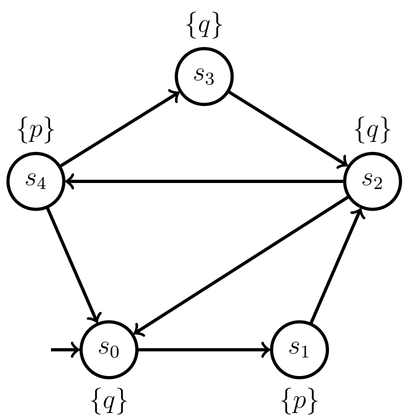
Use the ordering $b_0 < b_1 < b_2$ where b_0 is the most significant bit and b_2 is the least significant bit of the binary encoding.

(b) **2 bonus points:** Compute a BDD node for the set $P \cap Q$ using the BDD intersection algorithm (see below). Show the recursion tree.

> Recall the BDD intersection algorithm. Let B and C be two nodes of a multi-BDD. The node for the intersection of B and C is computed as follows:
>
> - If B and C are equal, then return B.
> - If B or C are the 1 leaf, then return the other BDD.
> - If B or C are the 0 leaf, then return 0.
> - Otherwise, compare the two variables labelling of B and C, and let x be the smaller among the two (or the one labelling both).
> - If B is labelled by x, then let B_1 and B_0 be the children of B; otherwise, let $B_1 := B$ and $B_0 := B$. Define C_1 and C_0 analogously.
> - Apply the strategy recursively to the pairs B_1, C_1 and B_0, C_0, yielding BDD nodes E and F. If $E = F$, return E, otherwise return $mk(x, E, F)$.

Question 5: Abstraction refinement (2 + 1 + 2 = 5 points)

Consider the Kripke structure $\mathcal{K} = (S, \rightarrow, r, AP, \nu)$ where $S = \{s_0, s_1, s_2, s_3, s_4\}$, $r = s_0$, $AP = \{p, q\}$, and $\rightarrow$ and ν are graphically represented as follows:



Let $\approx$ be the equivalence relation over S given by $s \approx t$ iff $\nu(s) = \nu(t)$.

(a) Construct the Kripke structure $\mathcal{K}'$ obtained by abstracting S with respect to $\approx$.

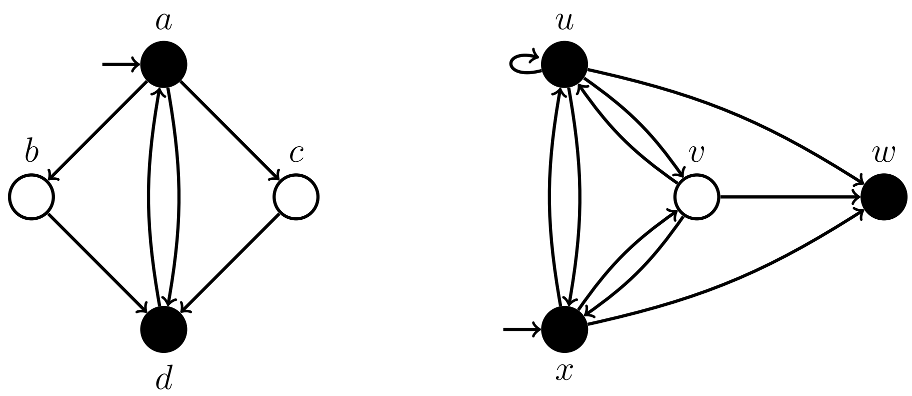
(b) Give a counterexample showing that $\mathcal{K}'$ does not satisfy $\mathbf{GF}p$.

(c) Following the procedure seen in class, use the counterexample to refine $\mathcal{K}'$ into a Kripke structure $\mathcal{K}''$.

(d) **2 bonus points**: Keep refining the abstraction until you prove that $\mathcal{K}$ satisfies $\mathbf{GF}p$.

Question 6: Simulations and bisimulations (2 + 2+ 2 = 6 points)

Consider the two following Kripke structures $\mathcal{K}_1$ (left) and $\mathcal{K}_2$ (right) over $AP = \{p\}$. States coloured black satisfy proposition p and others do not.



(a) Does $\mathcal{K}_2$ simulate $\mathcal{K}_1$? If your answer is *yes*, then give a simulation relation, and if it is *no*, then explain why no simulation relation exists.

(b) Does $\mathcal{K}_1$ simulate $\mathcal{K}_2$? If your answer is *yes*, then give a simulation relation, and if it is *no*, then explain why no simulation relation exists.

(c) Define what is a bisimulation. Give a Kripke structure $\mathcal{K}_3$ bisimilar to $\mathcal{K}_2$ but with fewer states than $\mathcal{K}_2$.

Question 7: Pushdown systems (3 + 3 + 1 = 7 points)

Consider the following recursive program with a global boolean variable `x`:

```
        boolean x;

        procedure foo;                 procedure bar;
f0:       x := not x;          b0:       if x then
                                           call foo;
f1:       if x then                      endif;
            call foo;
          else                 b1:       return;
            call bar;
          endif;

f2:       return;
```

(a) Model the program, where the value of `x` is not initialized, with a pushdown system $\mathcal{P} = (P, \Gamma, \Delta)$. Give explicit enumerations of the set of control states P, the stack alphabet Γ, and the set of rules Δ.
Hint: Δ contains 10 rules.

(b) Let E be the set of all configurations of $\mathcal{P}$ with empty stack. Give a $\mathcal{P}$-automaton recognizing the language E. Use the saturation rule to compute a $\mathcal{P}$-automaton recognizing the language $pre^*(E)$. For each transition added by the saturation rule, briefly explain how it is generated.
Hint: The $\mathcal{P}$-automaton for $pre^*(E)$ should have 10 transitions.

(c) Is there any configuration of $P \times \Gamma^*$ from which it is impossible to reach a configuration with empty stack? Briefly justify your answer.

Solution 1: LTL and Büchi automata (2 + 2 + 2 + 3 = 9 points)

$\phi_1 = \mathbf{FG}(p\ \mathbf{U}\ q)$ — eventually, $\emptyset$ must stop occurring and q must appear infinitely often.
$\phi_2 = \mathbf{FG}(\neg p \to q)$ — eventually always $p \vee q$.
$\phi_3 = \mathbf{G}(\neg p \vee (p\ \mathbf{R}\ q))$ — equivalent to $\mathbf{G}(\neg p \vee (p \wedge q))$.

(a) No. We have $p\ \mathbf{U}\ q \implies p \vee q$ and hence $\mathbf{FG}(p\ \mathbf{U}\ q) \implies \mathbf{FG}(p \vee q) \equiv \mathbf{FG}(\neg p \to q)$.

(b) Yes. $\{p\}^\omega$ satisfies ϕ_2 but not ϕ_1.

(c) Yes. $\{p, q\}^\omega$ satisfies all three.

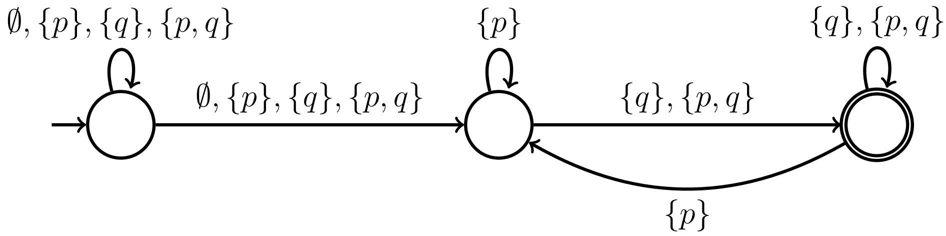
- (a) is satisfied because $\mathbf{G}(p \wedge q) \implies \mathbf{G}(p\ \mathbf{U}\ q)$;
- (b) is satisfied because $p \wedge q \implies p \vee q$; and
- (c) is satisfied because $\phi_3 \implies \mathbf{G}(\neg p \vee (p \wedge q))$ and the word ensures $p \wedge q$ at all points.

(d) It should accept

- $\{p, q\}^\omega$
- $\emptyset\{p, q\}^\omega$
- $\{p\}\{q\}^\omega$
- $(\{p\}\{q\})^\omega$
- $\{q\}^\omega$

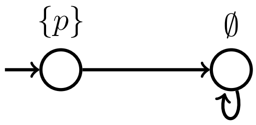
and it should reject

- $\emptyset^\omega$
- $\{p\}^\omega$

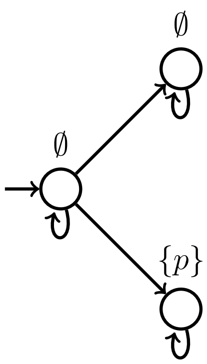


Solution 2: CTL (1 + 1 + 1 + 1 = 4 points)

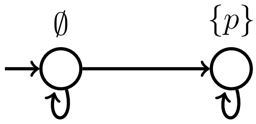
(a) $\mathbf{EF}p$ but not $\mathbf{EFAG}p$:



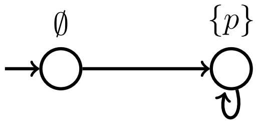
(b) $\mathbf{EFAG}p$ but not $\mathbf{AGEF}p$:



(c) **AGEF**p but not **AGAF**p:



(d) **AGAF**p but not **AG**p:



Solution 3: Partial order reduction (1 + 1 + 1 + 1 + 1 = 5 points)

(a) $I = \{(a,c),(c,a),(b,c),(c,b)\}$. Note that I cannot contain $\{(a,b),(b,a)\}$ since the diamond property is violated in s_3.

(b) $U = \{a\}$.

(c) No, C_1 is violated because b can be executed before a.

(d) No, C_2 is violated because b is visible.

(e) Yes:

(i) C_0 is satisfied because $red(s_2)$ is not empty.

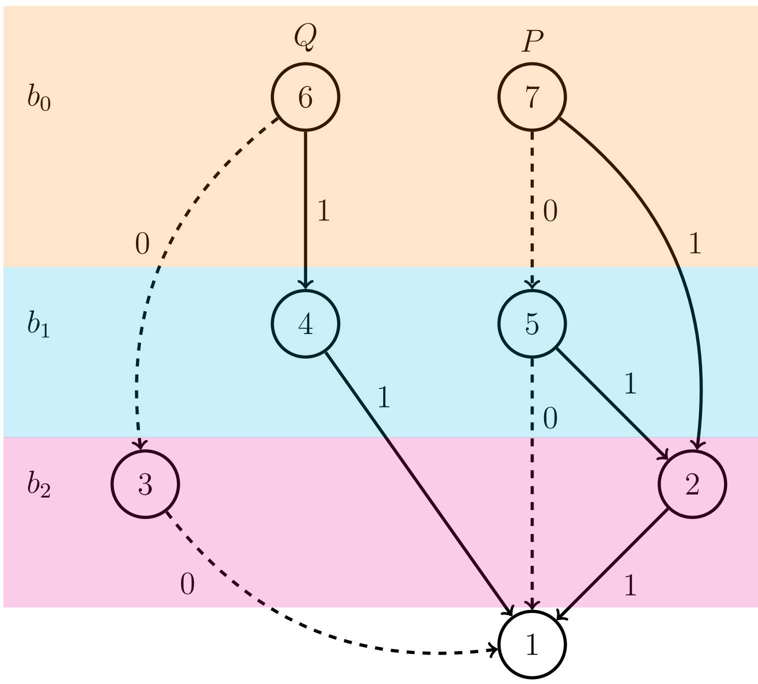
(ii) C_1 is satisfied because a is executed before b in all three paths starting in s_2.

(iii) C_2 is satisfied because a is invisible.

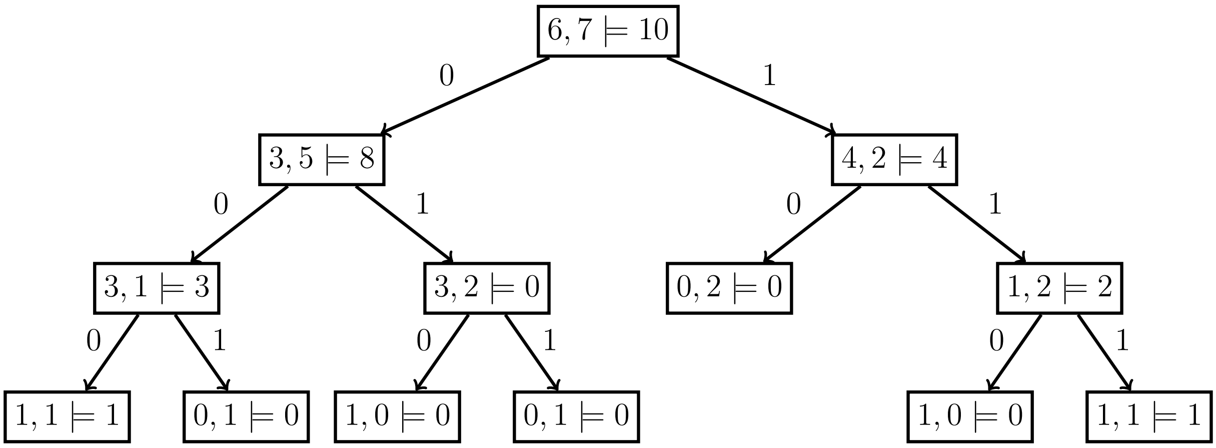
(iv) C_3 is satisfied because the only cycles of the reduced Kripke structure are the self-loops at s_3 and s_7, and $red(s_3) = en(s_3) = red(s_7) = en(s_7) = \{b\}$ which follows from C_0.

Solution 4: Binary decision diagrams (4 points)

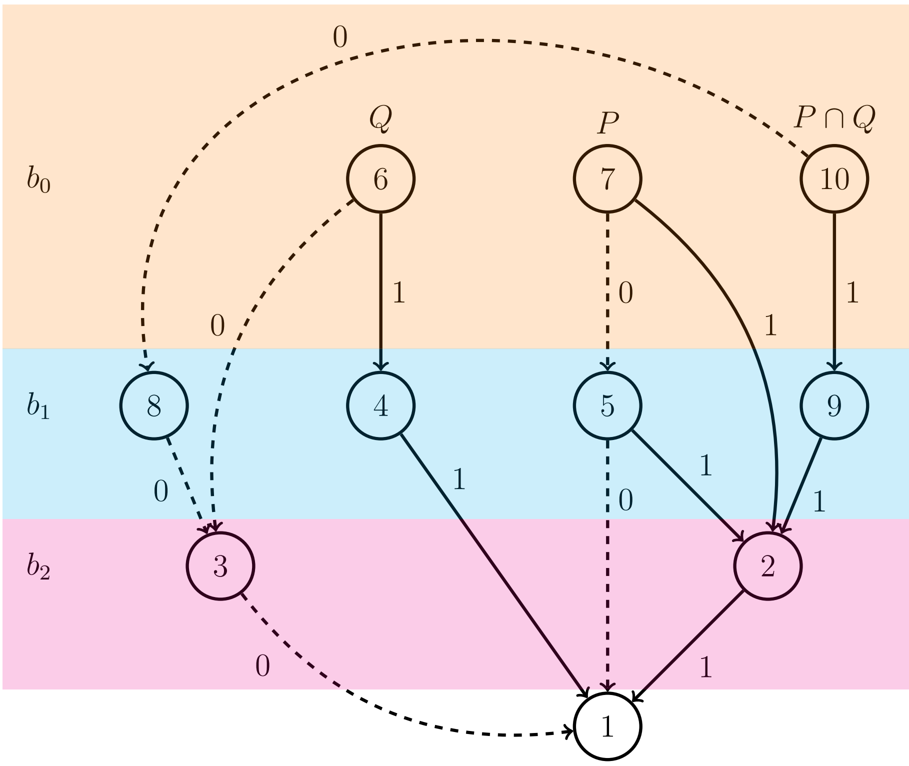
(a)



(b) Recursion tree:



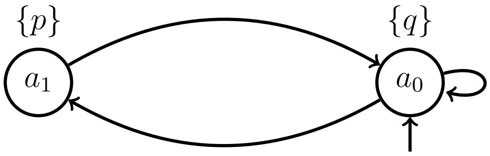
Resulting multi-BDD:



Solution 5: Abstraction refinement (2 + 1 + 2 = 5 points)

(a) First abstraction:

$$a_0 = \{s_0, s_2, s_3\},$$
$$a_1 = \{s_1, s_4\}$$



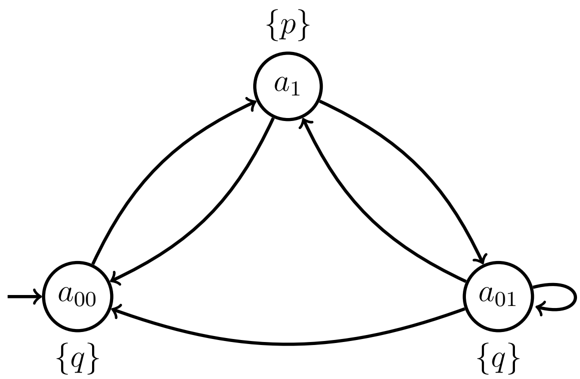
(b) Counter-example: ${a_0}^\omega$.

(c) We have $|a_0| = 3$, so we unroll the loop 4 times:

$$
\begin{tikzcd}
a_0 & a_0 & a_0 & a_0 \\
\longrightarrow s_0 & s_0 & s_0 & s_0 \\
s_2 \arrow[ur] & s_2 & s_2 & s_2 \\
s_3 \arrow[ur] & s_3 & s_3 & s_3
\end{tikzcd}
$$

Fails to concretize in 1 step, so we realize that we need to refine. The states which are reachable from the initial state should be distinguished from the states which still have successors. We introduce:

$$
\begin{aligned}
a_{00} &= \{s_0\}, \\
a_{01} &= \{s_2, s_3\}, \\
a_1 &= \{s_1, s_4\}.
\end{aligned}
$$



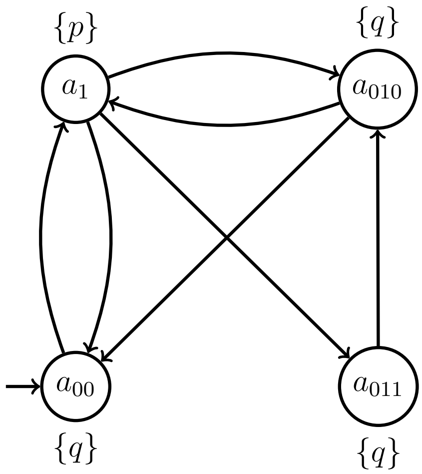
(d) New counter-example: $a_{00}a_1a_{01}a_{01}{}^{\omega}$:

$$
\begin{tikzcd}
a_{00} & a_1 & a_{01} & a_{01} & a_{01} & a_{01} \\
s_0 \arrow[r] & s_1 \arrow[r] & s_2 & s_2 & s_2 & s_2 \\
 & s_4 \arrow[r] & s_3 \arrow[ur] & s_3 & s_3 & s_3
\end{tikzcd}
$$

We split s_2 and s_3, and introduce:

$$
\begin{aligned}
a_{010} &= \{s_2\}, \\
a_{011} &= \{s_3\}.
\end{aligned}
$$

We obtain the following which satisfies $\mathbf{GF}p$:



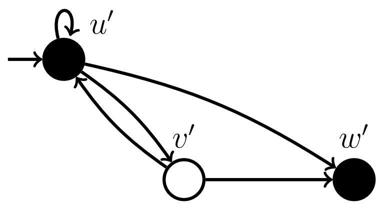
Solution 6: Simulations and bisimulations (2 + 2+ 2 = 6 points)

(a) Yes: $\{(a, x), (b, v), (c, v), (d, w)\}$.

(b) No, we prove it by contradiction. Assume that $\mathcal{K}_1$ simulates $\mathcal{K}_2$ and let H be the simulation. Since x and a are the respective initial states, $(x, a) \in H$. Since $(x, a) \in H$ and $x \to u$ where u is black, there must exist a black state in $\mathcal{K}_1$ with a transition from a. The only candidate in this case is d. Hence, $(u, d) \in H$. By a similar argument, if $(u, d) \in H$ and $u \to v$ where v is white, then there must exist a white state in $\mathcal{K}_1$ with a transition from d — which is not the case. Hence $\mathcal{K}_1$ does not simulate $\mathcal{K}_2$.

(c) A relation H is called a bisimulation between $\mathcal{K}$ and $\mathcal{K}'$ iff H is a simulation from $\mathcal{K}$ to $\mathcal{K}'$ and $\{(t, s) : (s, t) \in H\}$ is a simulation from $\mathcal{K}'$ to $\mathcal{K}$.

We merge x and u in $\mathcal{K}_2$ to obtain $\mathcal{K}_3$ which is as follows:

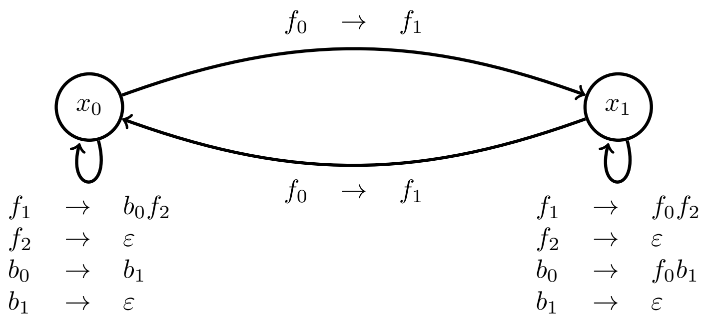


We define the bisimulation relation as follows:

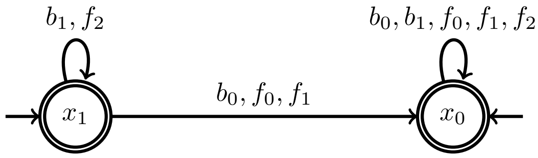
$$H = \{(x, u'), (u, u'), (v, v'), (w, w')\}.$$

Solution 7: Pushdown systems (3 + 3 + 1 = 7 points)

(a) The stack alphabet is $\Gamma = \{f_0, f_1, f_2, b_0, b_1\}$ and the pushdown system is as follows:



(b)



(c) No, there is no such configuration since the $\mathcal{P}$-automaton obtained in (b) accepts $P \times \Gamma^*$.